ESSAYS ON THE THEORY OF RATIONAL EXPECTATIONS EQUILIBRIUM

by

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ABSTRACT

This thesis examines the effect of relaxing the assumption of perfect information in a pure exchange economy. Two concepts of equilibrium are explored. The first, a non-cooperative concept, is that of a Rational Expectations Equilibrium. The second, a cooperative concept, is that of the core with differential information.

It is shown that cooperative equilibria always exist, while non-cooperative equilibria exist almost always. The two notions of equilibrium are then compared. It is shown that they need not coincide, as in the case with perfect information, but that they do so for almost all economies. Many examples are presented to illustrate the difference between perfect information and imperfect information equilibria.

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I. INTRODUCTION

A competitive, or Walrasian, equilibrium (as described in e.g. Debreu [1959]) provides a description of the allocation of resources in a non-cooperative economy in which two hypotheses are satisfied : (a) traders are endowed with perfect information, and (b) they take market prices as exogenously given. In recent years, there have been various attempts at deriving the second hypothesis, (b), from more basic considerations, e.g. Gabszewicz and Vial, [1972], Novshek and Sonnenschein, [1979], Hart [1979], and papers in a recent Journal of Economic Theory Symposium [1980]. The first hypothesis, that of perfect information is, however, usually taken as given. Almost all of these papers derive a competitive equilibrium as the limit of imperfect, or monopolistic, competition, as the relative size of any trader becomes infinitesimal.

This thesis is concerned with the implications of relaxing the hypothesis of perfect information.

Upon relaxing the first hypothesis, one has to modify the notion of equilibrium to deal explicitly with information and expectations. One concept of equilibrium, which retains the second hypothesis, is that of a rational expectations equilibrium (REE). Traders are allowed to have differential information, but a requirement of equilibrium is that traders expectations about relevant uncertain variables are fulfilled. This eliminates the first hypothesis, and imposes in its stead a restriction on equilibrium.

The relaxation of (b) and the definition of an REE lead immediately to a host of interesting problems. To list some, (i) Do REE's exist ? (ii) How is information disseminated ? (iii) Can agents use available information, together with some learning procedure, to correct expectations and attain an REE ? (iv) How much information is revealed at an REE ? (v) Can an REE be viewed as the outcome of a cooperative game played between traders ? (vi) Does an REE have any optimality properties ?

The first four questions have been analyzed by numerous authors, and a brief survey of the literature is presented in part 2 of this introduction. The thesis is divided into various sections, each of them concerned with analyzing Rational Expectations Equilibria (REE) in a model with differential and imperfect information in which prices reveal information. The main problem analyzed is whether the standard results on existence, optimality and core equivalence extend to such an economy.

We shall be concerned with questions (i), (ii), (iv), (v) and especially (vi). We ask whether a rational expectations equilibrium is a natural notion of equilibrium in an economy with differential and imperfect information in the sense that the properties of such an equilibrium are similar to those of a competitive equilibrium in the case of perfect information. The main properties examined are the following:

(a) Does an REE exist under the same assumptions as those required in the Walrasian model ?

(b) Is the core with differential information a well defined concept ? Do there exist core allocations ? How much information is required to sustain core allocations ?

(c) Does core equivalence hold ? In other words, are REE's Pareto optimal, and can Pareto optimal allocations be decentralized as REE's ?

The following is a summary of the chapters:

(1) Introduction. The introduction summarises the results of the thesis, and provides the motivation for the work.

(2) The model. This chapter describes the model used, and the assumptions employed during the course of the analysis. It also contains the statements of various mathematical results required in defining the model. For example, it contains a detailed discussion of the topologies used on the space of preferences, etc.

(3) Existence: In this chapter, it is proved that the equilibrium concept concept used, that of an REE, exists for almost all economies. The proof is demonstrated diagrammatically as well as formally. There is a discussion of how the existence result relates to similar results found in the literature.

(4) The Core with Differential Information. Chapter 4 examines the notion of a cooperative equilibrium in an economy with differential information. It is proved that the core in such an economy is well

defined, and some properties of the core are analyzed. In particular, we explore the relationship between this core and that in a perfect information economy.

(5) Rational Expectations Equilibria and the Core. In this chapter, we demonstrate the result that for almost all economies, core equivalence obtains. This means that generically, an REE lies in the core of an economy with differential information, and also that core allocations can almost always be decentralized as REE's. Core equivalence does not obtain for all economies, unlike the case with perfect information, and we provide examples to demonstrate some of the reasons for non-equivalence.

(6) Trading After the Occurrence of the State. Chapter VI investigates the problem of core equivalence in the case where all trading is ex-post. It is shown that some of the results of the previous chapters change significantly. In particular, it is shown that the problems of information revelation, non-existence and suboptimality arise only in the ex-ante case.

(7) Large Economies. In Chapter VII, we examine our results in the case where there are a large number of agents. We show that none of the results obtained in Chapters III-V are affected by the size of the economy.

(8) Enforceability of Contracts In Chapter VIII, we ask whether the contracts arrived at in the ex-ante trading model are enforceable in the sense that at least one agent would sue in case of breach. It is shown that core allocations are enforceable, but that REE's need not be. In the light of the results of Chapter V, we show that even if an economy reaches a sub-optimal REE ex-ante, such contracts will generally not be observed in practice, since there is an incentive to re-negotiate all such contracts.

The main difference between the economy we shall examine and the Walrasian model is that prices have to serve two functions: they have to ensure that markets clear, and also transmit information. We then get the following scenario. Agents have some initial information, and based on this information, they formulate their plans. This leads to an excess demand function, which depends on the initial information of all agents. A zero of this excess demand function, if it exists, leads to a price vector. Agents can then use the price vector to obtain more information than they initially possessed. This leads to a revised set of excess demands, and correspondingly, to a new equilibrium price vector. A fixed point of this process is called a Rational Expectations Equilibrium. An REE is thus a price vector, p, that satisfies the condition that based on the information contained in the price vector, consumer demands are such that markets clear at the same price vector. It is this element of simultaneity which leads to potential existence and optimality problems. Our first concern is existence. An REE does not always exist, due to a discontinuity caused by prices revealing information. In Chapter III, we shall prove that for almost all economies, there exists an REE. We require no additional assumptions besides those required for existence in the traditional Arrow-Debreu model. This is in contrast to the generic existence results obtained in the literature. In Chapter III, we point out why our result is a significant generalisation of available results, and also why it cannot be improved.

In Chapter IV, we take up the study of the core with differential information. Contrary to previous negative results on such cooperative equilibria, we demonstrate that the core with differential information is a well defined concept, examine some of its properties, and compare it to the core of an economy without any informational problems.

As indicated, we shall examine the core of this economy, and this leads us to consider possible cooperative outcomes in a game played by traders with imperfect and differential information. Wilson [1976] proposed two alternate definitions of the core for such an economy : the 'coarse' core, in which no information is disseminated, and the 'fine' core, in which all information is revealed. In the next section, we define our notion of the core, which lies in between these two cases, and discuss the various concepts. Kobayashi [1981] has recently formalised the coarse core in accordance with Aumann's [1976] definition of common knowledge.

The concept of the core we shall use is the following. A coalition is called viable if, for the members of the coalition, there is some reallocation of initial endowments together with some system of communication, such that this reallocation is individually rational given post-communication information. One restriction imposed is that post-communication information be at least as good as initial information. The core then consists of allocations not blocked by any viable coalition.

An REE is a non-cooperative equilibrium, and we shall compare it to the core, a cooperative equilibrium concept. We ask whether an REE is a natural concept of equilibrium in the sense that if agents were allowed to cooperate, they could not produce an allocation of resources dominating that at an REE. The motivation behind this is simple enough. One argument used to defend the concept of an REE is that if expectations are not rational, then there must be gains to be had by changing the currently held set of beliefs. This argument implies that an equilibrium in which rationality of expectations is not imposed must produce an allocation sub-optimal relative to that at an REE. One of the strongest justifications for analysing a competitive equilibrium is that they lie in the core, and core allocations are Pareto optimal. It is therefore natural to ask whether a similar justification holds for REE's, and this is the topic of Chapter V.

In the model we shall use, uncertainty is represented by the existence of alternate states of nature. Agents can possess differential and imperfect information. By "information" we mean the ability to distinguish between states of nature. This is the same as in Radner [1968].

The presence of imperfect information leads to constraints on trading. In particular, it is required that if an agent is unable to distinguish between states s_1 and s_2, then his trades in these states must be the same. Such informational constraints on trading were introduced by Radner [1968]. He showed that in such an economy, there will generally be a less than complete set of markets in operation. We are thus in the position of examining the optimality of rational expectations equilibrium in an incomplete market setting.

Hart [1975] examines the properties of equilibria when markets are incomplete. In his model, there is an exogenously specified set of operative markets, and he finds that some REE's may be suboptimal relative to others. Furthermore, he shows that the opening up of new markets can actually be welfare decreasing. The essential difference between the work of Hart and the analysis presented below is that in Hart [1975], the set of markets is exogenously specified, while here, the number of markets in operation is determined endogenously by the information held by agents. This leads to a different definition of a rational expectations equilibrium, since prices reveal information which can change the set of markets. An REE here is a price vector which confirms expectations and is compatible with the implied set of operative markets. This will become clearer below. This makes the analysis quite different, and during the course of the analysis, we point out some of the differences.

The analysis is divided into two parts. In Chapters II-V, the setting is an exchange economy where binding contracts are made prior to the occurrence of the state. The kinds of markets we are considering are thus contingent claims or insurance markets. The case in which trading takes place after a state of nature occurs leads to a considerably different analysis, and results for this case are presented in Chapter V.

To summarize the results for the ex-ante case, it is shown that :

(i) An REE need not exist

(ii) However, an REE exists for almost all economies

(iii) the core is non-empty

(iv) The core of this economy can differ from that of an Arrow-Debreu economy

(v) The core with differential information is closely related to that in an Arrow-Debreu economy with certain informational restrictions.

(vi) Core allocations need not be fully revealing

(vii) If an allocation lies in the core, then it also lies in the coarse core and the fine core

(viii) There may exist multiple REE's that can be Pareto ranked

(ix) An REE need not lie in the core

(x) There exist Pareto Optimal REE's that do not lie in the core.

(xi) A fully revealing REE lies in the core

(xii) For almost all economies, there exists an efficient REE

(xiii) An REE need not be fully revealing

(xiv) There exist core allocations which cannot be decentralised as REE's.

(xv) For almost all economies, a core allocation can be decentralised as an REE.

The ex-post case, in which trading takes place after a state of nature occurs, leads to a variety of modelling problems. In Chapter V, we therefore discuss, by means of examples, the approach taken here, which differs from the models of Radner [1979] and Shefrin [1979], who are concerned with similar problems. The thrust of our argument is that this case reduces essentially to that of competitive equilibrium under certainty, and accordingly, there is no sub-optimality associated with equilibrium. This depends crucially on how one models consumer behavior, so we discuss some alternatives, and compare our model to those existant in the literature. The main conclusion of this Chapter is that the problems of information revelation, non-existence and sub-optimality arise only in the ex-ante case, where decisions are made prior to the occurrence of the state.

In Chapter VII, we analyze how the results obtained in the previous chapters are affected by the size of the economy. We demonstrate that these results are invariant to the number of consumers. The setting for this section is a simplified version of the model of Hildenbrand [1974], with a non-atomic measure space of consumers. We show that all the results of the previous chapters hold in this case. In particular, REE's need not exist, but do so generically, the core is non-empty, and that the monopoly power due to an informational edge <u>does not</u> disappear with a large number of agents.

The final Chapter examines the issue of the enforceability of contracts. We show that REE's may not be enforceable. In particular, inefficient REE's will not be enforceable. It is shown that core allocations are enforceable.

References and diagrams are collected at the end of the thesis.

1.II Motivation and Related Literature

There are two sets of literature which deal with the concept of a Rational Expectations Equilibrium. The first is the macroeconomic literature, while the second concerns the microeconomic modelling of markets in situations of uncertainty. To motivate our analysis, we briefly distinguish between them, and indicate how our analysis is related to existing literature.

The hypothesis of Rational Expectations was introduced by Muth [1961], although the concept, if not the term, seems to have appeared earlier (see Modigliani and Grunberg [1956]). The concept was mainly ignored until Lucas [1972] used it in a far-reaching critique of both existing economic theory and econometric practice. The critique was of the so-called "natural rate hypothesis", or the Phillips curve, concerning the trade-off between inflation and unemployment (or real output). His argument was that the conventional treatment of expectations necessarily permitted "both short- and long-run Phillips-like tradeoffs between inflation and real output"(Lucas [1976]). In the words of Hall, "the benefits of inflation derive from the use of expansionary policy to trick economic agents into behaving in socially desirable ways even though their behavior is not in their own best interests ...... the gap between actual and expected inflation measures the extent of the treachery". In this context, the assumption of Rational Expectations reduces this gap to zero, i.e. agents are assumed to be able to correctly predict inflation, at least on average. This was illustrated by Lucas [1972], and somewhat more dramatically by Sargent and Wallace [1975]. The conclusion from the analysis was that if agents' expectations were rational in the above sense, then there is no exploitable trade-off, even in the short run. Subsequent developments in the macroeconomic literature have focused on the econometric implications of such an assumption (e.g. Barro [1977], Sargent [1976], Sims [1972]), and on the examination of problems whick weaken the above conclusion (e.g. Fischer [1978]).

The macroeconomic interest in expectations was followed by an attempt to integrate the concept of Rational Expectations into general equilibrium, microeconomic models. Previous attempts at dealing with uncertainty (e.g. Debreu [1959], Radner [1968]) had not dealt with the case of endogenous expectations. The introduction of the term, and its meaning, however, were somewhat different from the macroeconomic usage of the term. The definition of an REE focused on the relationship between information held by agents and the market price. It only required that this relationship, as perceived by agents, not be contradicted by observable market variables such as the equilibrium price. Thus, the micoeconomic definition imposes Rational Expectations as a condition of equilibrium, not as an assumption on individual behavior. Microeconomic models then posed questions of the form : do there exist such equilibria, how much information is revealed at such equilibria, etc. We remark that this is different from examining market outcomes when agents are assumed to have Rational Expectations. The micro usage of the term is therefore more limited than the macro usage.

The first sets of results obtained were mainly negative. Green [1977] and Kreps [1977] provided examples which showed that REE's need not exist. Radner [1979] demonstrated that within a specific model of asset trading, REE's would exist generically, and be fully revealing. Allen [1981] extended this result to a more general model, which is very similar to the model we shall employ. In Chapter III, we will provide an existence proof which, besides being more simple, is much more general than any of these.

Alongside the above developments, which were in the context of models with differential information, a theory of REE's with asymmetric information emerged. A survey is contained in a recent paper of Grossman [1982]. There is also a literature on learning and convergence to an REE, a survey being contained in Bray [1980].

The literature mentioned above deals with the existence issue. On the other hand, there has been a curious lack of study of the optimality properties of REE's. Further, even though market equilibria with differential information has been studied, there has been almost no attempt to extend alternate models of economic systems in such a setting. The only exception seems to be Wilson [1976], who examined the core of an economy with differential information, again with mainly negative results. In Chapter IV, we remedy this deficiency by developing a theory of cooperative equilibria in economies with differential information. In subsequent chapters, we examine the optimality properties of REE's.

II. THE MODEL

The economy is one of pure exchange, and consists of m traders or consumers, L goods and a finite set, $S=\{s_1,\ldots,s_k\}$, of alternative states of nature. All trade is conducted prior to the occurrence of the state of nature. The market, therefore, is one of contingent claims. Consumers are indexed by i, $i=1,\ldots,m$. Each consumer has an endowment stream, $\{w^i(s)\}$, where $w^i(s)$ is the endowment of consumer i in state s, $w^i(s)\gg 0$. In Chapter VII, we shall examine the case of a non-atomic measure space of consumers.

A consumers information is given by a partition $I^i=\{E_1,\ldots,E_n\}$. Each E_j is called an event. These events are subsets of S, $E_j\subset S$, are mutually exclusive, $E_i\cap E_j=\emptyset$ if $i\neq j$, and are exhaustive, $\cup_j E_j=S$. We denote by I^i_o the initial information of consumer i, i.e. his information prior to his entering the market. We shall assume that the only source of initial information is the endowment stream. This means that $w^i(s)$ is I^i_o-'measurable', i.e. s_1 and s_2 lie in the same event in I^i_o if and only if $w^i(s_1)=w^i(s_2)$. The 'if' part of this assumption is not needed for the analysis, but we impose it to emphasize that endowments are a source of information. We will need the requirement that initial information be compatible with initial endowments, i.e. that $s_1,s_2\in E_j\in I^i_o$ only if $w^i(s_1)=w^i(s_2)$. This means that if endowments are different across states s_1 and s_2, then the consumer will be able to tell them apart. Our results are not affected if consumers have more information than is contained in endowments.

Preferences

There are L commodities in each state of the world, and k alternative states. The commodity space is, therefore, a subset of R^{Lk}. Let X denote the non-negative orthant of R^{Lk}. X will be the consumption set of all consumers.

Note: We have assumed that the consumption set is the same for all consumers. This is purely for notational simplicity.

A preference relation on X is a reflexive and transitive binary relation $(\geq)$, which satisfies the following additional properties:

(i) Continuity: $\{ y : y(\geq)x \}$ is closed

(ii) Monotonicity: $y \geq x$, $y \neq x$ implies $y(>)x$.

(iii) Convexity: $\{ y : y(\geq)x \}$ is convex.

Each such preference relation defines a subset, F, of $X \times X$,

$F = \{ (x,y) : y(\geq)x \}$

As proposed by Kannai, [1974], we can topologize the space of preferences with the topology of closed convergence on the space of subsets of $X \times X$. See Hildenbrand, [1974], for a detailed description of this topology. Briefly, preferences are said to converge if the associated subsets, e.g. F, converge. Convergence of sets is defined as follows:

For a sequence of subsets F_n, let

$\liminf A_n = \{z$: there exists $z_n \to z$, with $z_n \in A_n$, all n$\}$

$\limsup A_n = \{z$: there exists $z_n \in A_n$, all n, and a further subsequence, $z_n^k \to z\}$

limsup A_n is then the set of all subsequential limits of sequences contained in A_n, while liminf A_n is the set of limits of all convergent sequences in A_n. Then,

$A_n \to A$ in the closed convergence topology if and only if liminf A_n = A = limsup A_n. Note that liminf $A_n \subset$ limsup A_n, so to prove that $A_n \to A$, it suffices to prove that limsup $A_n \subset A \subset$ liminf A_n This definition of convergence has some intuitive content, to which we now turn.

Consider the indifference curves shown in Figure 1, $(\geq)_n$ and $(\geq)$. The topology is defined in terms of the preferred-or-indifference sets, like the shaded area. Then, the preferences depicted by $(\geq)_n$ converge to those depicted by $(\geq)$ if the following two conditions are satisfied: (a) if $y_n(\geq)_n x$, all n, and $y_n \to y$, then $y(\geq)x$, and (b) if $y(\geq)x$, then there exists $y_n \to y$ such that $y_n(\geq)_n x$, all n. The interpretation, then, is that all the preferred-or-indifferent sets converge.

Let P denote the space of all preference orderings with the topology of closed convergence. The aggregate space of preference relations is the m-fold product of P, denoted by P^m. P is endowed with the topology of closed convergence, (see Hildenbrand [1974, p.18]). P^m is given the product topology. We shall denote by P^m_s the subset of P^m which consists of strictly convex preferences.

Theorem 1: With the topology of closed convergence, P^m becomes a separable, metrizable space.

Proof: See Hildenbrand [1974]

All the examples that we shall give will assume that consumers are expected utility maximizers. Further, since most other work in this area assumes that consumers are expected utility maximizers, we establish a notation for this case. We shall denote by $\rho^i(s)$ the subjective probability of state s occurring, as held by consumer i. In this case, we shall write the expected utility function as

$$V^i(x) = \Sigma\, \rho^i(s)U^i(x(s)).$$

We remark that the hypothesis of expected utility maximization is not needed for any of the results established.

Prices are elements of R^{Lk}. We denote by Δ° the set of all strictly positive prices. These prices will generally impart some information to consumers. Therefore, we denote by I^i the final information of trader i. We shall further assume that prices are the only source of information to consumers besides initial endowments. A formalisation is provided below, after the following technicalities.

For two partitions I and J, we say that I is finer than J written $I \subset J$, if for $A \in I$, $B \in J$, either $A \subset B$ or $A \cap B = \emptyset$ In this case, J is said to be coarser than J, and I is also said to refine J. The meet of I and J, written as $I \vee J$, is defined as the coarsest partition that refines both I and J. For example, if $I = \{(a,b),(c,d)\}$ and $J = \{(a,c),(b,d)\}$, then $I \vee J = \{(a),(b),(c),(d)\}$. $I \vee J$ can thus be thought of as the best information obtained from pooling the information in I and J. The notation $\vee I^i$, all i, will denote the meet of all the I^i, as i ranges over all the agents.

<u>Definition 1:</u> Given prices p and initial information I^i_o, $i=1,\ldots,m$, I^i is a <u>final information</u> if

(i) $I^i \subset I^i_o$, all i.

(ii) $\vee I^j \supset I^i$, all i.

(iii) s_1, $s_2 \in E_j \in I^i$ if and only if $s_1, s_2 \in E_k \in I^i_o$ <u>and</u> $p(s_1)=p(s_2)$.

(i) states that final information can be no worse than initial information, (ii) states that final information cannot be finer than (better than) that available in the economy as a whole, while (iii) ensures that prices and endowments are the only sources of information. An alternate, but equivalent, way to define final information is the following. Let K(p) be the partition of S generated by p, i.e. $s_1, s_2 \in E \in K(p)$ if and only if $p(s_1)=p(s_2)$. Then, $I^i = I^i_o \vee K(p)$. We maintain the form of the definition for ease of comparison with final information in the cooperative economy, in which case there are no prices.

Faced with prices p and final information I^i, a consumers budget set is given by :

$$\beta(p, w^i, I^i) = \{x \in X: \Sigma\, p(s).x(s) \leq \Sigma\, p(s).w^i(s)$$
$$\text{and } x(s_1)=x(s_2) \text{ if } s_1, s_2 \in E_j \in I^i\}.$$

Lack of information thus imposes a constraint on trades open to the consumer. In particular, if he cannot distinguish between two states of the world, then his trades conditional on those two states must be the same. In other words, if a traders final information does not allow him to distinguish between states s_1 and s_2, then he cannot promise to deliver different commodity bundles in the two states. He is only allowed to make trades which he can actually verify. Without such an assumption, a severe moral hazard problem can arise, with corresponding implications for the enforceability of such contracts. Such informational constraints on trading were introduced by Radner, [1968], and a more detailed discussion of these constraints can be found there.

We have incorporated the informational constraints directly into the budget set rather than as a separate set of constraints. Therefore, the budget set actually represents the set of feasible consumption bundles for the representative consumer, both in the sense of affordability and in the sense of informational feasibility.

Since the informational constraints apply to all consumers, we assume that aggregate initial information is perfect, i.e. that $\vee I^i_o=[\{s_1\},\ldots\ldots,\{s_k\}]$ We do this because if two states cannot be initially distinguished by anybody, then we can effectively view them as a single state. This assumption also rules out a conceptual problem that can arise in this model if there are multiple equilibria, which is that the equilibrium price may reveal more information than is available in the economy as a whole (see Kreps, [1974]).

A consumers problem is to choose a $(\geq)_i$-maximal element of $\beta(p,w^i,I^i)$. The possibility of non-existence arises because β is not upper-hemi continuous in p. Next, we define a rational expectations equilibrium.

Definition 2: A Rational Expectations Equilibrium (REE) is a price vector p, a set of final informations $\{I^i\}$, and a set of allocations $\{x^i(s)\}$, such that

(i) $(x^i(s))$ is a $(\geq)_i$-maximal element of $\beta(p, w^i, I^i)$

(ii) $\Sigma\, x^i(s) \leq \Sigma\, w^i(s)$, all s.

The fact that we require I^i to be a final information, together with condition (i), implies that the information on which demands are based is the same as the resulting final information, which means that expectations are fulfilled.

Such an equilibrium can be thought of as a fixed point of the following iterative process. Agents start with information I^i_o, observe some initial prices p_o, and formulate their excess demands. These are transmitted to the market, and an equilibrium price, say p_1, is established. At this point, agents have more information, since they can use p_1 to update I^i_o. In particular, if $p(s_1) \neq p(s_2)$, then they can distinguish between s_1 and s_2. Let I^i_1 denote their information having observed p_1, $I^i_1 \subset I^i_o$. This leads to a new set of excess demands, and consequently a new set of prices, say p_2, new information $I^i_2 \subset I^i_o$, and so on. Note that it is not necessary that I^i_2 be finer than I^i_1. At a fixed point of this process, the information revealed by prices is the same as that which led to these prices, i.e. is an REE. We note that the definition does not imply that differential information cannot prevail at an REE. For example, if $I^1 = [\{a,b\},\{c,d\}]$ and $I^2 = [\{a,b,c,d\}]$, then $p(a)=p(b)=p(c)=p(d)$ satisfies (iii), but agent 1 has strictly more information than agent 2. Next, we demonstrate that an REE need not be fully revealing.

<u>Example 1 : (A non-revealing REE)</u>

The initial data are :

		state a	state b	state c
agent	1	5	3	3
agent	2	3	5	3
agent	3	4	4	6

$I^1=[\{a\},\{b,c\}]$

$I^2=[\{a,c\},\{b\}]$

$I^3=[\{a,b\},\{c\}]$

$\rho^i(s)=1/3$, all i,s; $U^i(x)=\log(x)$, all i.

Then, $p(a)=p(b)=p(c)=1$ is a non-revealing REE for this economy, as is easily checked by computing informationally constrained excess demands and computing equilibrium prices.

III. EXISTENCE

In general, an REE need not exist, due to an informational discontinuity (see Radner [1979]. The model used by Radner is an ex-post model, however, and in the following example, it is shown that this problem arises in the ex-ante case. In Chapter V, we will show that if the ex-post case is modelled differently from Radner [1979], there is no existence problem.

Example 2 : (Non-existence of an REE)

Consider an economy with two agents, three states and one commodity. Initial information and endowments are given below.

	state a	state b
agent 1	2	2
2	1	3

$I^1=[\{a,b\}]$

$I^2=[\{a\},\{b\}]$

$U^1(x) = U^2(x) = \log(x)$; the subjective probabilities of the agents are $(\alpha,1-\alpha)$, $(\beta,1-\beta)$.

Suppose there is a fully revealing equilibrium, i.e. one with $p(a) \neq p(b)=1$. Then, agent 2 can distinguish between states a and b, and the constraint $x^2(a) = x^2(b)$ imposed by initial information no longer holds.

Let Y_1 and Y_2 denote the incomes of the two agents. Then, market clearing for the good in state a requires :

$$\alpha Y_1/p(a) + \beta Y_2/p(a) = 3,$$

while that for the good in state b requires

$$(1-\alpha)Y_1+(1-\beta)Y_2=5.$$

It is easy to see that for any choice of α and β such that $\alpha+\beta=3/4$, the only equilibrium is $p(a)=1$, which means that agent 1 cannot distinguish between states a and b, and the informational constraint once again becomes binding. We conclude that there is no fully revealing equilibrium for this economy.

Suppose, then, that we look for an equilibrium with $p(a)=p(b)=1$.

Agent 2 is still unconstrained, and his demands thus stay the same; agent 1, however, is restricted to consume the same amount in both a and b. The resulting excess demand functions are :

$$(1/2)Y_1+\beta Y_2 = 3 \text{ and}$$
$$(1/2)Y_1 + (1-\beta)Y_2 = 5.$$

For $\beta\neq 1/4$, there is no solution. Thus, there is no equilibrium with $p(a)=p(b)$. We conclude that there in no REE for this economy for appropriate α and β.

It is instructive to examine the source of non-existence in more detail. The usual method of proving the existence of equilibrium employs a fixed point theorem. For example, Brouwer's fixed-point theorem states that any continuous function from a compact, convex and non-empty subset of R^n to itself has a fixed point. The relevant functions that arises in this type of model can fail to be continuous (upper- hemicontinuous in the case of correspondences), and this happens because a small change in prices can lead to a drastic change in an agents' feasible set. Figure 2 illustrates. Suppose e(a)=e(b). Then, if p(a) ≠ p(b), the feasible set is the area ODE. If p(a)=p(b), the feasible area 'shrinks' suddenly to the line OA. Thus, a small change in prices can lead to a discontinuous change in the feasible set. The budget correspondence thus fails to be upper-hemicontinuous, and demand functions therefore need not be continuous. In general, therefore, an equilibrium need not exist. Figure 3 depicts an economy for which there is no REE.

It is interesting to compare the non-existence of equilibrium here with that in Hart [1975], who uses a different model. Hart's example of non-existence demonstrates that for certain market structures, an equilibrium may fail to exist. Non-existence is a possibility even if all agents have identical information. However, the imposition of a uniform upper bound on trades rules out non-existence. Differential information, and the revelation of information, however, can change the set of operative markets, leading to non-existence, as shown by the example. Here, it is seen that if we endogenise the market structure, there may be <u>no</u> set of markets which give rise to an equilibrium. The imposition of an upper bound on net trades can always yield the existence of a no-trade equilibrium (e.g. by setting the bound on net trades to be zero), and we do not discuss this further.

The first question one might ask is whether a small perturbation in initial endowments can restore equilibrium in the example. The answer is clearly yes, since we can always perturb endowments so as to give agent 1 perfect information. In this case, there is no informational problem, and the economy reduces to an Arrow-Debreu economy, and there exists an equilibrium.

Such perturbations of endowments change initial information. The question of interest is whether there exist changes in endowments compatible with a given set of initial information which restores equilibrium in the model. This leads to two possible modelling strategies. The first is to define the space of endowments of each agent so that endowments are compatible with the given set of initial information. The second, easier, way is to consider perturbations in <u>income.</u> Income perturbations get 'added-on' to the end of the budget constraint, and do not interfere with initial information. We shall follow the second strategy. Thus, we shall examine whether small changes in an agents income restores equilibrium in the model. It is clear that for given prices, we can always convert income into an informationally compatible endowment. Such perturbations in income (endowments) will be combined with perturbations in preferences.

It is seen from Figure 3 that there is a small change in preferences such that the full information equilibrium is not at a non-revealing price. In this case, point E is an REE, and a small change in preferences restores equilibrium to the example. This example demonstrates the following simple fact. Suppose we let consumers choose consumption plans without imposing any informational constraints. This economy will have an equilibrium under our assumptions, as is well known. Furthermore, if this equilibrium is at a fully revealing price, i.e. $p(s_1) \neq p(s_2)$ if $s_1 \neq s_2$, then it is also be an REE with final informations equal to VI^i_o. Thus, if we can prove that most unconstrained economies have fully revealing equilibria, then we have proved that most economies have an REE.

This is not a new approach. Previous analyses of this problem have followed the same reasoning, e.g. Radner [1979] and Allen, [1981] Both these authors assume that agents are expected utility maximizers, and Radner proves generic existence by perturbing subjective probabilities. Allen uses log-linear perturbations of expected utility functions. We too shall prove generic existence by perturbing preferences. The proof, however, is different from that employed by either of these authors, and the result proved is stronger. Some differences are :

(i) We do not impose any restrictions on the relative dimensionalities of the set of states and the set of commodities.

(ii) We do not require preferences to be differentiable or strictly convex.

(iii) We do not require the existence of selections from the Walrasian correspondence.

(iv) The proof is very simple, as will be shown below.

Prior to a formal presentation of the proof, we describe the technique we shall use, and compare it to those used in the literature.

Consider a full information economy, i.e. an economy in which there are no informational constraints. Such an economy is a standard Walrasian economy, and under the assumptions we have made, there exists an equilibrium for this economy. Let $W((\geq),w)$ denote the Walrasian correspondence. It is well known that W is closed and non-empty valued, and also that it is upper hemi-continuous. Next, let us examine the price space. In R^2, it is easy to see that almost all prices are fully revealing, as in Figure 4. Thus, any non-revealing price vector can be derived as the limit of a sequence of fully revealing price vectors. Further, the set of all fully revealing prices is open. An immediate consequence of the upper hemi-continuity of W is that the set of preferences for which there exists a fully revealing full information equilibrium is open. It remains to establish density, and this has been the focus of, e.g Allen, [1981] and Radner, [1979].

These authors have focused on the invertibility of the 'price function'. Their analysis proceeds as follows. Suppose there is one commodity and k securities. Let y denote information which is available in the economy as a whole, and $\pi_j(y)=\text{Prob}(s=s_j|y)$. Let $P(\pi)$ denote the equilibrium price vector given π Then, if P is invertible, knowing P is equivalent to knowing π, i.e. s. Allen shows that $P(\pi)$ is generically invertible. The proof, however, requires strong regularity assumptions. In particular, it requires that P be a smooth function. This means that there must exist a (globally) smooth selection from the equilibrium correspondence. The literature on the local uniqueness of equilibrium demonstrates that all one can hope for is a <u>locally</u> smooth selection. It is unclear what restrictions the selection assumption imposes on preferences and endowments. We shall not require any such assumption.

Next, we provide a diagrammatic exposition of the proof. We know that if the full information equilibrium is fully revealing, then it is an REE. Suppose there does not exist an REE. The full information equilibrium must, therefore, be non-revealing. Consider the representative consumer, depicted in Figure 5. $x(p)$ is his full information demand at prices p, and p is a non-revealing price. We know that there exists another price vector, q, arbitrarily close to p, such that q is fully revealing. Further, we know that markets clear at $x(p)$. Suppose, then, that we give the consumer an income perturbation, k, such that $x(p)$ is just feasible at prices q. This leads to a new budget set, denoted by the dotted line. Finally, we perturb his indifference curve, U, to the dashed indifference curve, U^q, such that $x(p)$ is actually chosen at prices q. Then, q is a full information, fully revealing equilibrium for the perturbed economy. As q gets closer to p, k becomes smaller, and U^q gets closer to U. A formalisation of this simple idea is provided below.

We start with some definitions and elementary results.

Definition 3: A full information equilibrium is a price vector p and allocations $(x^i(s))$ such that :

(i) $x^i(s)$ is $(\geq)_i$-maximal on $\{x : \Sigma\, p(s).x(s) \leq \Sigma\, p(s).w^i(s)\}$
(ii) $\Sigma x^i(s) \leq \Sigma_i w^i(s)$, all s.

Definition 4: A price vector is called fully revealing if $s_1 \neq s_2$ implies $p(s_1) \neq p(s_2)$.

Lemma 1: If there exists a fully revealing full information equilibrium, then there exists an REE.

Proof: In this case, let $I^i = VI^i_o$. Since p is fully revealing, I^i is a final information. Then,

$\beta(p, w^i, I^i) = \{x : p.x \leq p.w^i\}$. The full information allocation is therefore feasible, and markets clear. Further, it satisfies condition (ii) of the definition of an REE.

Lemma 2: If the full information equilibrium occurs at autarky, then there exists an REE.

Proof: Let p be the full-information equilibrium price such that each agent chooses to consume his endowment. To each agent, assign final information as follows. Let K be the partition of S generated by p, i.e. $s_1, s_2 \varepsilon E \varepsilon K$ if and only if $p(s_1) = p(s_2)$. Let $I^i = I^i_o VK$. It is easily checked that I^i is a final information. The initial endowment satisfies all measurability requirements, and is feasible at prices p. Thus, p is also an REE.

It follows that for there to not exist an REE, the full information equilibrium must involve trade and must be at a non-revealing point. In what follows, endowments will be held fixed at $w^i(s)$. We shall denote by k^i the income perturbation of consumer i, and the full information budget set will then be denoted by $\beta^i(p,k^i)=\{ x : p.x \leq p.w^i+k^i\}$

Let $\phi: P^m \to \Delta^0 \times Y$ be the full information equilibrium correspondence. To each element of P^m and to each income perturbation $k \in Y$, it associates a set of full information equilibrium prices. As is well known, ϕ is non-empty valued under the assumptions we have made.

Definition 5: A correspondence $F:X \to Y$ is said to be upper-hemi continuous (uhc) at x if $F(x) \neq \emptyset$, and if for every neighbourhood V of F(x), there exists a neighbourhood U of x such that $F(z) \subset V$ for all $z \varepsilon U$. If F is uhc at all x in X, then it is simply called upper-hemi continuous, abbreviated to uhc.

Definition 6: A correspondence $F:X \to Y$ is call lower-hemi continuous (lhc) at x if $F(x) \neq \emptyset$, and $x_n \to x$, $y \varepsilon F(x)$ implies there exists a sequence y_n, $y_n \to y$ with $y_n \varepsilon y$.

If a correspondence is both uhc and lhc, then it is said to be continuous.

Lemma 3: ϕ is uhc.

Proof: See Hildenbrand [1970], or Hildenbrand and Mertens [1972].

Lemma 4: $\{p\varepsilon\Delta^0 : p$ is fully revealing$\}$ contains an open and dense subset.

Proof: Let p be a fully revealing price. Thus, $p(s_i) \neq p(s_j)$ for $i \neq j$. Let $\delta = \min \{ |p_j(s_1) - p_i(s_2)| : p_j(s_1) \neq p_i(s_2)\}$ where $s_1 \neq s_2$, and $i,j=1,\ldots,m$. Then, $\delta > 0$. Pick $0 < \varepsilon < \delta$ Then, any p' in the ε-neighborhood of p is fully revealing. This establishes openness.

To prove density, let p be a non-revealing price. Thus, $p(s_1)=p(s_2)$ for some $s_1 \neq s_2$. Choose k numbers, $\varepsilon_1,\ldots,\varepsilon_k$, $\varepsilon_i \neq \varepsilon_j$, and define $p_n(s)$ by:

If $p(s_1) \neq p(s')$, all $s'\varepsilon S$, $p_n(s_1)=p(s)$.
If $p(s_1)=p(s_2)$ for some s_2, $p_n(s_1)=p(s_1)+(1/n)\varepsilon_s$.
The ε_j can be chosen such that $p_n(s_1) \neq p_n(s_2)$ all $s_1 \neq s_2$.
Then, $p_n(s) \rightarrow p(s)$, all s, and $p_n(s)$ is fully revealing for all n.

Theorem 2: The subset of P^m for which there exists a fully revealing full information equilibrium is open.

Proof: ϕ is uhc. From Lemma 4, there is an open subset of Δ^0, say A, such that every price in A is fully revealing. Then, Definition 5 implies that the set of preferences in P^m for which there exists a fully revealing equilibrium is open.

Let $\psi: \Delta^0 \rightarrow P_s^m \times Y$ be the inverse map induced by ϕ, but restricted to the set of strictly convex preferences. To each price p, ψ associates the set of preferences and income perturbations for which, given endowments w, p arises as the full-information equilibrium. The rest of the proof consists of establishing that ψ is lower hemi-continuous, which allows us to prove density. Then, the uhc property of ϕ establishes openness.

Lemma 5: ψ is non-empty valued.

Proof: Choose the income perturbation of each agent to be zero. Let $p \varepsilon \Delta^{0}$ be given. For each i, this defines a budget set, $\beta(p,w^{i})$. We shall construct preferences for each individual consumer, and we thus ignore the superscript i.

Let $(\geq)$ be any preference relation in P_{s}, and at prices p, let x(p) be the (unique) choice of consumption. Define a new preference relation, $(\geq)'$, as follows :

$y(\geq)'z \iff y-(w-x(p))\ (\geq)\ z-(w-x(p))$. It is easily seen that $(\geq)'$ is reflexive and transitive, and is also closed. We show that it is strictly convex and monotonic.

(i) $(\geq)'$ is strictly convex :

Let $y_1\ (\geq)'\ z$, $y_2\ (\geq)'\ z$.

Then, $y_1-(w-x(p))\ (\geq)\ z-(w-x(p))$, and $y_2-(w-x(p))\ (\geq)\ z-(w-x(p))$.

Since $(\geq)$ is strictly convex, we have that for $0<\lambda<1$,

$\lambda y_1+(1-\lambda)y_2-(w-x(p))\ (>)\ z-(w-x(p))$, and so $\lambda y_1+(1-\lambda)y_2\ (>)'\ z$.

(ii) $(\geq)'$ is monotonic : Let $y\ (\geq)'\ z$, and $y' \varepsilon X$, with $y' \geq y$. Then, we have that $y'-(w-x(p))\ (\geq)\ y-(w-x(p))\ (\geq)'\ z-(w-x(p))$, and so $y'\ (\geq)'\ z$.

Thus, $(\geq)' \varepsilon P_s$. Next, we show that at prices p, the $(\geq)'$-maximal element of the budget set is w, the endowment point. This is seen as follows.

Suppose not. Then, there exists $y' \varepsilon \beta(p,w)$ such that $y'\ (>)'\ w$, i.e. $y'-w+x(p)\ (>)\ x(p)$, which contradicts the fact that x(p) was chosen at prices p and preferences $(\geq)$, since $y'-w+x(p)$ is feasible at prices p.

Since each consumer chooses not to trade at prices p, given preferences $(\geq)'$, p is an equilibrium price for the set of preferences $(\geq)'$.

<u>Theorem 3:</u> Let $(\geq) \epsilon P_s^m$, and suppose that there is no REE for $(\geq)$. Then, there exists a sequence of preferences, $(\geq)_n \to (\geq)$, and a sequence of income perturbations, $k_n^i \to 0$, such that $(\geq)_n$ has a fully revealing full information equilibrium when consumers are given income perturbations k_n.

<u>Proof:</u> Our construction will be for an arbitrary consumer, and we therefore omit the superscript i. Let p be the full information equilibrium price. From Lemma's 1 and 2, we know that p is a non-revealing price, and also that there is trade at prices p. From

Lemma 4, we know that there exists a sequence, $p_n \to p$ such that each p_n is fully revealing. We shall construct a set of income perturbations, k_n, and preferences, $(\geq)_n$ such that p_n is the full information equilibrium for $(\geq)_n$, with $(\geq)_n \to (\geq)$, and $k_n \to 0$.

$w(s)$ is the endowment stream for the representative consumer, and at prices p, his income is $p.w = \Sigma p(s).w(s)$. Let

$k_n = p_n.x(p) - p_n.w$, where $x(p)$ is the unique[1] consumption bundle of the agent at prices p. We know that markets clear at prices p, i.e. that $\Sigma x^i = 0$. Then, $x(p)$ is feasible at prices p_n, with additional income k_n, since

$$p_n.x(p) = p_n.w + k_n.$$

Note that $p \gg 0$, since preferences are monotonic, and therefore we can choose all the p_n to be strictly positive.

Define a new set of preferences, $(\geq)_n$, by:

$$x \ (\geq)_n \ y \quad \text{if and only if}$$

$$x(p)+[(p_n/p)(x-x(p))](\geq)x(p)+[(p_n/p)(y-x(p))],$$

where $p_n x/p$ is the vector whose j'th element is $p_n^j x_j/p^j$, i.e. the multiplication and division is done element by element. Note that this

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[1] $x(p)$ is unique since $(\geq)$ is strictly convex.

construction of preferences depends on the initial preferences relation, $(\geq)$, and on the prices p and p_n.

It is straightforward to verify that $(\geq)_n$ is a preference relation on X, given $(\geq)$, p and p_n.

We claim that at prices p_n, x(p) will be chosen by the representative consumer. This is seen as follows. Suppose not. x(p) is feasible at prices p_n, so there must exist a feasible consumption vector, say y, such that $y(>)_n x(p)$. By construction of $(\geq)_n$, this means that $x(p)+[(p_n/p)(y-x(p))](>)x(p)$. But $x(p)+[(p_n/p)(y-x(p))]$ is feasible at prices p, since $p_n.y=p_n.w+k_n=p_n.x(p)$. This means that either $x(p)=y$, or that x(p) is not the optimal choice of consumption at prices p, a contradiction.

We already know that markets clear at x, and x(p) is thus a full information equilibrium at prices p_n. We need only to verify that $k_n \rightarrow 0$, and that $(\geq)_n \rightarrow (\geq)$.

Since $p_n \rightarrow p$, and $k_n=p_n.x(p)-p.w=p_n.x(p)-p.x(p)$, we get that $k_n \rightarrow 0$.

To demonstrate convergence of preferences, we shall denote by F the graph of $(\geq)$, and by F_n that of $(\geq)_n$. We have to show that $F_n \rightarrow F$ in the topology of closed convergence.

Thus, let $(x,y)\epsilon X$. To show that liminf $F_n=F$, let $x_n=x(p)+[(p_n/p)(x-x(p))]$, and $y_n=x(p)+[(p_n/p)(y-x(p))]$. Then, $p_n\to p$ implies that $x_n\to x$ and $y_n\to y$.

To show that limsup $F_n=F$, let $(x_n,y_n)\epsilon F_n$, all n, with $x_n\to x$, $y_n\to y$. Then, there exist $(x',y')\epsilon F$ such that $x'=x(p)+[(p_n/p)(x-x(p))]$ and $y'=x(p)+[(p_n/p)(y-x(p))]$. Since F is closed, we have that $(x,y)\epsilon F$, since $x'\to x$, $y'\to y$.

This concludes the proof.

Theorem 4: For an open and dense subset of $P^m\times Y$, there exists an REE.

Proof: Openness was established in Theorem 2. Density is a consequence of Theorem 4, since any convex preference relation is the limit of a sequence of strictly convex preference relations. This last part is proved by Kannai, [1974].

At this point, two remarks are in order. First, Theorem 3 establishes a property of the Walrasian correspondence, ϕ, which may be of independent interest. This is that the inverse map induced by ϕ is lower hemi-continuous (in fact, this inverse mapping is a continuous correspondence). Note that this does not imply that ϕ is continuous. An implication of this result is that small changes in relative prices can be obtained by small perturbations in preferences/incomes. This should be interpreted with care. It does not mean that small changes in preferences/incomes will lead to small changes in equilibrium prices, but only that there exists some perturbation of preferences/incomes leading to a small change in in prices.

The second remark concerns the existence of REE's when the set of potential markets is incomplete. In the model considered, we have k alternative states of nature, and accordingly, k potential markets for contingent commodities. The existant literature on the existence of REE's is sometimes concerned with the problem when there are less than k potential securities markets. Suppose that, for some reason, only $n<k$ such markets are allowed to operate. Then, p cannot reveal all the states in S. However, it is straightforward to verify that if there exists a full information equilibrium with the $n<k$ markets, then there generically exists an REE which fully reveals the n states in which trading is permitted. For this reason, the existence result established does not require the imposition of any restrictions on the number of operative markets.

The final part of this chapter is devoted to a discussion of the importance of the assumption that there are only a finite number of states. We shall argue that this assumption is crucial in the proof of generic existence because of a simple property of the price space. Green [1977] and Radner and Jordan [1977] have provided examples to show that with an infinite number of alternative states of the world, REE's do not exist generically. However, their analysis does not make clear the reason for this result.

Suppose, then, that there are an infinite number of alternative states of the world. To fix ideas, let $S=[0,1]$. The discussion will focus on the space of prices. Recall (Lemma 4) that the proof of Theorem 3 was based on the fact that "most" price vectors were fully revealing, and we could therefore approximate a non-revealing price by a fully revealing price.

With $S=[0,1]$, the price space becomes infinite dimensional. In each state of the world, there are L goods. This means that prices are functions, $p:S\rightarrow R_{+}^{L}$, with $p(s)\in R^{L}$ being the price vector if s. The price space, therefore, is the space of all functions from S to R_{+}^{L}. Let us denote this space by $\Delta(S,R_{+}^{L})$.

With a finite number of states, Δ^{0} was given the Euclidean topology. With a continuum of states, we have a choice of imposing a uniform topology. However, the point we wish to make is most clearly illustrated with the product topology or the topology of pointwise convergence. This topology is defined as follows.

Given a point $s\in S$ and an open set U in S, let

$B(s,U)=\{\ p\ :\ p\in\Delta(S,R_{+}^{L})\text{ and }p(s)\in U\}$

The sets $B(s,U)$ form a sub-basis for the product topology on $\Delta(S,R_{+}^{L})$. This means that the general basis element for this topology is a finite intersection of sets of the form $B(s,U)$. For $p\in\Delta(S,R_{+}^{L})$, therefore, a neighborhood consists of functions q which are "close" to p at finitely many points. Thus, q can be arbitrarily close to p, and still be non-revealing. This immediately implies that the set of all fully revealing prices in $\Delta(S,R_{+}^{L})$ cannot be open ; furthermore, it cannot contain an open set all of whose elements are fully revealing. This, in turn, implies that "most" price functions are not fully revealing, and that there is no direct proof that the space of economies with fully revealing equilibria is open. Green's example shows that, in fact, there is an open set of economies for which there is no REE.

The statement that accompanies examples of non-existence with a large number of states usually says that this is because the price space is "in some sense" too sparse to fully reveal information. The above discussion makes this notion precise.

IV. THE CORE WITH DIFFERENTIAL INFORMATION

In examining possible cooperative outcomes when traders have differential information, one has to allow for the possibility of information transfer. To start with, we can examine two polar cases : that with no information transfer, and that with all information being transferred. Wilson [1976] examined these cases, which go by the names of 'coarse core' and 'fine core' respectively. We shall examine these cases, and also a more general notion of the core.

The first and foremost problem we have to face is that of modelling the transfer of information across agents. In the non-cooperative market economy, prices served this purpose. There is no such obvious mechanism in the cooperative game. Further, established economic theory does not provide an adequate framework for modelling communication across agents in this setup. There have been two types of attempts at modelling a process of communication within an economy, and we discuss them briefly.

The first literature deals with the informational efficiency of the competitive mechanism, e.g. Mount and Reiter, [1974], Walker, [1977] and in a context more similar to ours, Jordan, [1976]. This literature demonstrates that the competitive mechanism is uniquely informationally efficient within the class of allocation mechanisms considered. These papers do not deal with cooperative games or with differential information, and it is unclear whether the methodology developed therein can be extended to accomodate such games.

The second literature examines information transfer in a principal-agent problem, e.g. Green and Stokey, [1980]. The principal receives information, and transmits it to the agent. The situation is one of conflict, not cooperation. Related work includes that of Marschak and Miyasawa, [1968].

We shall follow a different strategy. In the cooperative game, we shall assume that communication takes place within coalitions. However, we shall abstract away completely from method of communication, i.e. we will not impose any restrictions whatsoever on the process of information transfer. Thus, agents will be allowed to communicate by word of mouth, via quantity signals, etc. The rationale for following such a strategy is two-fold.

The first is the analogy with the traditional model of the core. In such models, it not necessary to know how coalitions form, or how agents within coalitions communicate to block allocations. It is possible to make definitive statements about the core without needing to know the process of coalition formation. A second such analogy is provided by our model of rational expectations equilibrium. There too it is possible to obtain results about REE's without needing to know how market prices are formed or how they get to contain information. The point is that it is possible to analyse the outcome of a game without having to model the process which leads to the outcome.

The second rationale comes from the results. We will show that the core in our case is very similar to that in the standard Arrow-Debreu economy.

The strategy, then, is to impose restrictions on final information and final allocations without specifying how they are attained. We will amend the traditional notion of the core by defining the core with differential information to be the set of allocation-information pairs which are not blocked by any coalition. A formal treatment follows.

Contracts are assumed to be binding, and we therefore require that the allocations of members of the coalition are compatible with information attained within the coalition. Thus, if $e^i(s_1) = e^i(s_2)$ but $\sigma^i(s_1) \neq \sigma^i(s_2)$, where σ^i is the allocation to agent i in the coalition, then the agent can distinguish between s_1 and s_2. Whether such contracts are enforceable is discussed in Chapter VII.

As before, lack of information poses a constraint on allowable trades. If a consumer is unable to distinguish between s_1 and s_2, then his trades conditional on s_1 must be the same as those conditional on s_2. In this Chapter, we will not assume that aggregate initial information is perfect. When necessary, this assumption will be stated as a hypothesis of a result.

Definition 7 : A coalition consists of a set of agents $C \subset \{1,\ldots,m\}$, a set of allocations $(\sigma^i(s))$, $i \in C$, $s \in S$, and a set of final informations (J^i), $i \in C$, satisfying :

(i) $J^i \subset I^i_o$, $J^i \supset \vee I^i_o$, all $i \in C$

(ii) $(s_1,s_2) \in A^i \in J^i$ implies $\sigma^i(s_1) = \sigma^i(s_2)$, all i

(iii) $\Sigma\sigma^i(s) \leq \Sigma e^i(s)$, all s

(iv) $\sigma^i \ (\geq)_i \ e^i$

(ii) is the 'measurability' constraint : if two states are not distinguishable, then the allocations in these states must be the same. Note the the converse need not hold.

(iii) requires that allocations be feasible within the coalition, while (iv) requires core allocations to be individually rational.

(i) requires that final information be no worse than initial information, and also that final information cannot be better than that obtained by pooling all the information available within the coalition. This is the only restriction imposed on information, and is in keeping with the usual definition of a coalition, which requires a coalition to be self-sufficient. An alternative condition would be to require that final information be no better that that available in the economy as a whole:

(i)' $J^i \subset I^i_o$, $J^i \supset \vee I^i_o$, $i=1,\ldots,m$.

This would allow coalition C to observe the actions of agents outside C, or to communicate with them, and infer more information than is available within C.

Our theorems on the existence of core allocations and the optimality of REE's are not affected if we replace (i) by (i)'. However, the size of the core is affected by this condition. We shall show that the core with (i) is generally larger than that with (i)', as one would expect.

Whether we choose to impose (i) or (i)' depends on our interpretation of the core. In particular, it brings up the question of the enforceability of contracts in a setting of differential and imperfect information. In the standard formulation, i.e. in an economy with perfect information, a coalition is required to be self-sufficient. It can only block, or improve upon, allocations based on its own resources, uncontingent upon the actions of agents not in the coalition. Condition (i) reflects this idea exactly. It requires a coalition to be self sufficient on its own, both in terms of physical feasibility of allocations and informational feasibility.

It is also in keeping with the individual informational constraints on trading, namely that agents can undertake only those trades which they can verify. Condition (i) extends this requirement to coalitions. For these reasons, we retain (i) as opposed to (i)' in our definition of a coalition.

Definition 8 : An allocation-information list (y^i, I^i) is blocked if there exists a coalition C with (σ^i, J^i) satisfying conditions (i)-(iii) of Definition 1 with $\sigma^i \ (\geq)_i \ y^i$ for all i in C, with strict preference holding for at least one i in C.

Note that we are employing a 'weak' notion of blocking; this is not important.

Definition 9 : An allocation (x^i) together with information (K^i), $i = 1,\ldots,m$ is said lie in the core if

(i) $K^i \supset \vee I^i_o$, $K^i \subset I^i_o$, all i

(ii) $\Sigma x^i(s) \leq \Sigma e^i(s)$, all s, and

(iii) (x^i, K^i), $i = 1,\ldots,m$ is not blocked.

The notion of optimality embodied in the core is different from that in Hart [1975] in that we allow information to change endogenously, while he defines optimality for fixed information.

Definition 10 : The coarse core consists of all allocations (y^i) which are not blocked by any coalition C, subject to the restriction that $J^i = I^i_o$, all i in C, and such that $\Sigma y^i(s) \leq \Sigma e^i(s$

Definition 11 : The fine core consists of all allocations (y^i) which lie in the coarse core, and are not blocked by any coalition C, with informations $J^i = \vee I^i_o$.

The definitions of the coarse core and the fine core are adapted from Wilson [1976]. (Wilson examines the case in which trading takes place after a state of nature occurs).

Definition 12 : A trading process is said to be fully revealing if $I^i = \vee I^i_o$, all i, where I^i is the information of agent i at the end of the trading process. If a trading process is not fully revealing, it is said to be non-revealing. Information I is said to be perfect[1] if every event in I contains only one element, i.e. $I=[\{s_1\},\ldots,\{s_n\}]$

Lemma 6 : If an allocation lies in the core, then it lies in both the fine core and the coarse core.

Wilson [1976] examines the core of a similar economy, in which trading is carried out ex-post. The problem that arises with his model also applies to the ex-ante case, so we examine it next. He considers two polar cases, the coarse core, with no communication, and the fine core, with full communication. He provides an example in which the fine core is empty. This, however, is not a problem in our model, and accordingly, we analyse his example to point out the differences.

Example 3 :

Initial endowments and information are :

	state		
	a	b	c
agent 1	5	1	3

[1] Note the distinction between full information and perfect information.

2	3	5	1
3	1	3	5

$I^1=[\{a\},\{b,c\}]$

$I^2=[\{a,c\},\{b\}]$

$I^3=[\{c\},\{a,b\}]$

$p^i(s)=1/3$, all i,s, and each agent has the same strictly concave, monotonic utility function in each state.

Note that in this example, initial information is not compatible with initial endowments. Thus, if agent 1 receives 3 units of the good, he will not know whether b or c has occurred. However, Wilson does impose measurability of the consumption bundle with respect to final information.

The example then proceeds as follows. Any two-agent coalition can achieve full information, and it is easy to see that with full information, the only possible allocation in the fine core is the initial endowment. Without any communication, i.e. when all agents retain their initial information after trading, the following allocation with no transfer of information dominates the initial endowment for sufficiently small ε :

	state		
	a	b	c
agent 1	$5+2\varepsilon$	$2-\varepsilon$	$2-\varepsilon$
2	$2-\varepsilon$	$5+2\varepsilon$	$2-\varepsilon$
3	$2-\varepsilon$	$2-\varepsilon$	$5+2\varepsilon$

Thus, there is no unblocked allocation with full information, i.e. the fine core is empty.

If we impose the requirement that initial information be compatible with initial endowments, the problem disappears. This is because the only states agents 2 and 3 will be unable to distinguish will have the same endowment, and, given measurability of the final allocation, the same utility. For example, if we modify endowments to :

	state		
	a	b	c
agent 1	5	1	1
2	3	5	3
3	1	1	5

then the initial endowment does lie in the fine core.

Theorem 5 : The core is non-empty

Proof : Let I be the coarsest partition that refines all the I^i_o. I then is the best information attainable, given initial informations. (I is called the meet of the I^i_o). Suppose I contains k (disjoint) events, indexed by j, j=1,...,k.
Note that for $(s_1,s_2) \varepsilon A_j \varepsilon I$, $e^i = e^i(s_2)$ for all i.

Next, we construct a perfect information economy with $k \le L$ goods, giving everyone initial information I. Endowments in the constructed economy are :

$e'^{i}_{j}(s) = e^{i}(s), \quad s \in A_j$. If agents are expected utility maximizers, then preferences are

$V'^{i} = \Sigma(\Sigma \; \rho^{i}(s))U^{i}(x_j)$.

with the obvious extension to the case of state-dependent preferences. The k-good economy is a standard exchange economy with perfect information, and by standard theorems, the core of this economy is non-empty.

The proof of the theorem is completed by noting that any allocation that is feasible in the original economy <u>and</u> is compatible with the information available in the original economy is also feasible in the constructed perfect information economy, and conversely. Thus, any allocation lying in the core of the constructed economy is also unblocked in the original economy, since otherwise, the blocking coalition could block that allocation in the constructed economy. Thus, the core of this economy is non-empty.

<u>Corollary 1 :</u> The coarse core is non-empty; the fine core is non-empty.

The proof of Theorem 1 consists essentially of demonstrating the existence of an unblocked allocation when everyone has full information. Next, we demonstrate that the core of this economy is generally different from that of the same economy with perfect information, and also that core allocations need not be fully revealing, in the sense that it is possible to achieve unblocked allocations without everyone having full information. We shall call perfect information equilibria Arrow-Debreu allocations.

Example 4 : (Arrow-Debreu allocations need not lie in the core) :

Consider an economy with two agents, one commodity and two states, where initial information and endowments are given in the following table.

	state a	state b
agent 1	4	4
2	1	1

$I^1=[\{a,b\}]$

$I^2=[\{a,b\}]$

Suppose $\rho^1=(1/3,2/3)$, and $\rho^2=(2/3,1/3)$. Then, the initial endowment lies in the core, but is not an Arrow-Debreu allocation for any strictly concave utility function. For example, $(4-\varepsilon\ ,4+\varepsilon\ )$, $(1+\varepsilon\ ,1-\varepsilon\ )$ dominates the initial endowment,but cannot be achieved since it requires more information than is available within the economy. Thus, core allocations need not be Arrow-Debreu allocations. On the other hand, an Arrow-Debreu allocation will always have different consumption in the two states, and given the availability of information, will always be informationally infeasible, so that Arrow-Debreu allocations need not lie in the core. Note that this example does not satisfy the assumption that aggregate initial information be perfect.

Next, we observe that core allocations need not be fully revealing.

<u>Example 5 : (A non-revealing core allocation)</u>

Consider the one-commodity, three state and three agent economy with initial data :

		state a	state b	state c
agent	1	5	3	3
	2	3	5	3
	3	4	4	6

$I^1=[\{a\},\{b,c\}]$

$I^2=[\{a,c\},\{b\}]$

$I^3=[\{a,b\},\{c\}]$

$U^i(x)=\log(x)$, all i. $\rho^i(s)=1/3$, all i,s.

Consider the allocation

$$\sigma^1=(11/3,11/3,11/3)$$

$$\sigma^2=(11/3,11/3,11/3)$$

$\sigma^3=(14/3,14/3,14/3)$

yielding

$V^1=\log(11/3)$; $V^2=\log(11/3)$; $V^3=\log(14/3)$.

Then, (σ^i) is compatible with the null information set, and is a core allocation. This is checked by first observing that no single agent coalition can block . Next, if we hold V^3 at log(14/3) and maximise the sum V^1+V^2, we find that log(11/3)+log(11/3) cannot be dominated. Thus, the grand coalition cannot block σ. Similarly, no two-agent coalition can either, and this allocation thus lies in the core. We do not present the computations, since they are straightforward. σ is then a non-revealing allocation in the core.

Next, we compare the core with differential information to that of the associated perfect information economy. We shall provide a complete characterization of core allocations. In view of Example 4, such a comparison will be carried out for the case in which aggregate initial information is perfect.

Let

$C^D = \{x \in R^{Lkm}$: there exist J^i, $i=1,\ldots,m$ such that (x^i,J^i) lie in the core $\}$

The superscript D stands for "differential information".

Definition 13 : The core of the associated perfect information economy is defined by:

$C^P = \{x \in R^{Lkm}: \Sigma x^i \leq \Sigma e^i$, $i=1,\ldots,m$, and there is no subset of agents, C, and an allocation σ, with $\Sigma\sigma^i(s) \leq \Sigma e^i(s), i \in C$, with $\sigma^i (\geq)_i x^i$, all $i \in C$ with strict preference holding for at least one $i \in C.\}$

C^P is seen to be the usual definition of the core.

Lemma 7 : If aggregate initial information is perfect, $C^P \subset C^D$.

Proof: Follows from the proof of Theorem 5.

Theorem 6 : If initial information is perfect, then $C^P = C^D$.

Proof: Let $x \in C^D$, and suppose x is not in C^P. Then, there exists a coalition C and an allocation σ which blocks x. Initial information is perfect, so σ^i is informationally feasible for all agents. Then, C with (σ^i, I_o^i) blocks x, i.e. x is not in C^D, a contradiction.
Together with Lemma 7, this completes the proof.

This Theorem provides the second rationale for our modelling strategy, as discussed at the beginning of this Chapter.

Definition 14 : An allocation x is said to be Pareto Optimal (PO) if:

(i) $\Sigma\, x^i(s) \leq \Sigma\, e^i(s)$

(ii) there is no allocation y, satisfying (i), with $y^i (\geq)_i x^i$, all i and $y^i (>) x^i$, some i

(iii) $x^i (\geq) e^i$, all i.

Note that we are defining PO allocations without reference to information. This is in contrast to Radner [1968]. We are also requiring them to be individually rational.

The following is well known.

Lemma 8 : If $x \in C^P$, then x is PO.

With differential information, we have:

Lemma 9 : If aggregate initial information is perfect, the $x \in C^D$ implies that x is PO.

Proof: If not, then the coalition consisting of all agents in the economy can block x. This grand coalition has perfect information.

Lemmas 8 and 9 lead to:

Theorem 7: If aggregate initial information is perfect, C^P and C^D can differ only over the set of PO allocations.

At this point, it makes a difference whether condition (i) or (i)' is imposed in the definition of a coalition. With (i)', coalitions are allowed to block on the basis of economy wide information. If aggregate initial information is perfect, (i)' leads to the conclusion that $C^P = C^D$. With (i), however, we have:

Theorem 8 : $C^P \neq C^D$, even if aggregate initial information is perfect.

The proof is by construction of an example which is given below (Example 6), after the following discussion.

We already know (Example 4) that if aggregate initial information is not perfect, then $C^P \neq C^D$. If VI^i_o is perfect, then we know that C^P and C^D can only differ over the set of PO allocations. To prove Theorem 8, therefore, we have to construct an example of a PO allocation that does not lie in C^P, but does so in C^D. This is alwaays possible, since C^P does not necessarily coincide with the set of PO allocations. The following example is based on this fact.

Example 6 :

Initial endowments and informations are:

		state a	state b
agent	1	2	2
agent	2	1	1
agent	3	1	1.1

$I^1=[\{a,b\}]$

$I^2=[\{a,b\}]$

$I^3=[\{a\},\{b\}]$

$\rho^1(a)=2/3,\ \rho^2(a)=1/3,\ \rho^3(a)=1/2.$

$U^i(x) = \log(x)$, $i=1,2,3$.

The data in the example have been chosen to satisfy the requirement that with perfect information, agents 1 and 2 have greater trading opportunities than either (1,3) or (2,3), and that without perfect information, agents 1 and 2 have no trading opportunity.

It can be verified that, within rounding error, the allocation σ, with

$$\sigma^1 = (2.51, 1.28)$$
$$\sigma^2 = (0.64, 1.31)$$
$$\sigma^3 = (0.85, 1.5)$$

is PO, but is blocked by a coalition of agents 1 and 2, and the allocation

$$x^1 = (2.38, 1.28)$$
$$x^2 = (0.62, 1.52).$$

However, σ is not blocked by either (1,3) or by (2,3).

With differential information, the trading opportunities of (1,3) and (2,3) remain the same as if they had perfect information, so it follows from above that (1,3) and (2,3) cannot block σ. Further, the grand coalition cannot block σ, as σ is PO. It can be calculated that (1,2) get greater utility with σ than under autarky. Thus, $\sigma \epsilon C^D$, but σ is not in C^P. Note that aggregate initial information in this example is perfect.

In the case of perfect aggregate initial information, we then have a complete characterization of allocations in C^D. They are allocations in C^P together with those PO allocations which are blocked by perfect information coalitions and these coalitions require more information than they possess to block these allocations in the case with differential information.

This leads to the following remark. We observe that in the Example, agent 3 is, in general, better off than in the associated perfect information game, since it is not possible for agents 1 and 2 to collude against him. As a result, one would expect the outcome of such a game to be biased towards agent 3, due to his informational edge. He is in the position that without him, agents 1 and 2 cannot trade at all, and they can always gain from trade if he is included. This suggests a line of research which we shall not endeavor to pursue here. In Chapter VII, we will show that this type of informational advantage does not disappear even if there are a large number of agents.

V. RATIONAL EXPECTATIONS EQUILIBRIA AND THE CORE

In this Chapter, we compare the notions of equilibrium developed in Chapters II-IV. We are interested in analyzing if core equivalence holds for our economy. We shall re-impose the assumption that aggregate initial information is perfect.

From the proof of Theorem 5 (Chapter IV), it is clear that any fully revealing REE must lie in the core. The reasoning behind this is that a fully revealing REE is just the standard competitive equilibrium for the constructed perfect information economy, which, by standard theorems, lies in the core of the constructed perfect information economy. The proof shows that any such allocation lies in the core of the original economy, and a fully revealing REE must therefore lie in the core.

Theorem 9 : A fully revealing REE lies in the core.

Proof : We prove this result for the case of expected utility maximization. The proof for the more general case of state dependent utility follows directly.

Let $I = \vee I_o^i$, and let $\{(p(s),(x^i(s)),I\}$ be a fully revealing REE. Suppose the theorem is false, i.e. there exists a viable coalition C which blocks $(x^i(s))$. Then, for at least one i in C,

$V^i(\sigma^i) > V^i(x^i)$, where σ^i is the allocation to agent i given by the coalition. This implies that σ^i is not feasible for agent i at prices p(s) and information I, since otherwise he would have chosen σ^i instead of x^i. Since $I=VI_o^i$, σ^i cannot be informationally infeasible, and so it must be the case that

$$p.\sigma^i > p.e^i \text{ for at least one } i, \text{ and}$$

$$p.\sigma^i \geq p.e^i \text{ for all } i \text{ in } C.$$

However, $\Sigma\, \sigma^i \leq \Sigma\, e^i$, which implies that

$p.\Sigma\, \sigma^i \leq p.\Sigma\, e^i$, which leads to the usual contradiction.

This leads us to ask whether non-revealing REE's can lie in the core, and the answer to this is yes, as the following example shows.

Example 7 : (A non-revealing equilibrium in the core)

We continue with example 5 of Chapter IV.

		state		
		a	b	c
	1	5	3	3
agent	2	3	5	3
	3	4	4	6

$I^1=[\{a\},\{b,c\}]$

$I^2=[\{a,c\},\{b\}]$

$I^3=[\{a,b\},\{c\}]$

$\rho^i(s)=1/3$, all i,s. $U^i(x)=\log(x)$, all i.

From example 1, we know that $p_1=p_2=p_3=1$ is a non-revealing REE for this economy. At these prices, equilibrium allocations are :

$$x^1=(11/3,11/3,11/3)$$
$$x^2=(11/3,11/3,11/3)$$
$$x^3=(14/3,14/3,14/3)$$

From example 5, we know that this allocation lies in the core. This is then a non-revealing REE in the core.

In general, however, an REE need not lie in the core, and core allocations cannot always be decentralised as REE's. The first assertion is easy to illustrate diagrammatically, and is shown in Figure 6.

Example 8 : (Efficient and inefficient REE's)

The Edgeworth-Bowley box in the Figure shows an economy with two agents, two states and one commodity. Agent 1 has the null information set, while agent 2 has perfect information. The initial endowment point, E, is an REE for this economy. No information is revealed to agent 1, and the informational constraint thus forces him to trade along his 45 line. E is the utility maximising point for agent 2 at prices $p_1 = p_2$, as depicted. However, any point in the shaded lens-shaped area dominates E, and E is thus an inefficient REE for this economy.

It is easy to assign numerical values to realise the situation depicted in the Figure. Note that there also exists an efficient, fully revealing equilibrium for this economy. This points out that there may exist multiple REE's, which can be Pareto-ranked. Note that if all agents have the same initial information, then an REE always lies in the core. This is in contrast to the example of Hart [1975], where the sub-optimality arises due to sequence constraints. There are no sequence constraints in our model.

This example also shows that this type of inefficiency is invariant to a re-distribution of income via lump-sum transfers. In the Figure, all such transfers lie along the 45 degree line of agent 1. For any such transfer of income, there exists an inefficient REE. Of course, one can always perturb endowments so as to make this a perfect information economy, and if all perturbations are allowed, then an REE will be generically efficient (since the subset of the space of endowments which does not give perfect information is of lower dimensionality than the entire space of endowments, and hence of (Lebesgue) measure zero).

On the other hand, the following theorem can be proved:

Theorem 10 : Suppose $((\geq),0)\epsilon P^m \times Y$ has an inefficient REE. Then, there exist $(\geq)_n \to (\geq)$, and $k_n \to 0$ such that $((\geq)_n, k_n)$ has an REE that lies in the core, all n.

Proof: From Theorem 9, we know that the full information price vector for $((\geq),0)$ must be non-revealing. From Theorem 3 and Lemma 1, there exist $(\geq)_n$ and k_n with $k_n \to 0$, $(\geq)_n \to (\geq)$, such that each $((\geq)_n, k_n)$ has a fully revealing REE.

Corollary: For an open and dense subset of $P^m \times Y$, there exists an efficient REE.

In the previous Chapter, we examined the relationship between C^P and C^D, core allocations with perfect and differential information. We have seen that fully revealing REE's lie in C^P (Theorem 9), and Example 8 depicts an REE which does not lie in C^D. The next example is perhaps somewhat more interesting. It constructs an REE that is Pareto Optimal, non-revealing and not in C^D.

Example 10 : (Pareto Optimal REE's need not lie in the core).

We examine a three-consumer economy. There are two states of the world, a and b, and one good in each state of the world. Initial data are :

		state a	state b
	1	1	2
agent	2	2	1
	3	5	5

$I^1=[\{a\},\{b\}]$

$I^2=[\{a\},\{b\}]$

$I^3=[\{a,b\}]$

$\rho^1(a)=2/3,\ \rho^2(a)=1/3,\ \rho^3(a)=2/5.$

$U^i(x) = \log(x),\ i=1,2,3.$

Then, it can be verified that the allocation x, with

$x^1=(2,1)$

$x^2=(1,2)$

$x^3=(5,5)$

is Pareto Optimal. It is the result of maximizing

$\lambda_1 V^1+\lambda_2 V^2+(1-\lambda_1-\lambda_2)V^3$,

obtained with weights $\lambda_1=\lambda_2=.16215$.

The allocation x does not lie in the core. A coalition of agents 1 and 3 blocks it with perfect final information and consumption

$\sigma^1=(2.069,1.045)$

$\sigma^2=(3.93,5.955)$

Next, consider the non-cooperative case. It is easily verified that $p(a)=p(b)=1$ is a non-revealing price which, together with final informations equal to I_o^i and allocations equal to x, forms an REE.

The conclusion, so far, is that most economies have an efficient REE. However, there exist REE's which are Pareto Optimal that do not lie in the core, as well as REE's that do not lie in the core and are not Pareto Optimal.

It has not proved possible to answer the following question: is it true that for most economies, all REE's will lie in the core ? This is because the properties of the equilibrium correspondences we have to work with are not strong enough to generate this type of result, as we now elucidate.

We know (Lemma 4, Theorems 2 and 3) that "most" prices are fully revealing, that any strictly positive price vector can be obtained as the result of utility maximization, and that this inverse relation (the mapping ψ) is continuous. To establish that <u>all</u> price equilibria are fully revealing, a stronger result is needed, namely, that every closed subset of Δ^0 can be an equilibrium price set for some economy. In terms of notation, we have that $\psi:\Delta^0 \rightarrow P^m \times Y$ is non-empty valued. But this only establishes that any single price vector can be obtained as the full information equilibrium via utility maximization. If there are multiple equilibria, we need to say something about sets of equilibrium prices. Letting 2^Δ be the set of all closed subsets of Δ^0, we could denote by $\theta:2^\Delta \rightarrow P^m \times Y$ the map that associates to each <u>subset,</u> A, of Δ^0 the set of preferences/incomes that lead to A as being the set of equilibrium prices. Then, $(\geq),k \in \theta(A)$ if and only if $p \in A$ implies that $p \in \phi((\geq),k)$, i.e. every element of A is a full information equilibrium for $(\geq),k$. We only know that this is true at every point p. The result would require that θ be non-empty valued, and that θ be a continuous correspondence. It is not clear whether this requirement is satisfied, or even true.

Next, we take up the question of whether core allocations can be decentralized as REE's. By "decentralizing an REE" we mean: does there exist a set of lump-sum transfers and a price vector such that the allocation in the core is attained as an REE ?

To construct an example of a core allocation which cannot be decentralised as an REE, we simply have to observe that the core is always non-empty while an REE need not exist. Thus, if we have an example of an economy in which there is no REE, we immediately have core allocations which cannot be decentralised.

Example 9 : (A core allocation which cannot be decentralised)

We take the initial data from example 2 of Chapter III.

	state a	state b
agent 1	2	2
2	1	3

$I^1 = [\{a,b\}]$

$I^2 = [\{a\},\{b\}]$

$\rho^1 = (\alpha, 1-\alpha)$; $\rho^2 = (\beta, 1-\beta)$; $U^i(x) = \log(x)$, all i.

Then, the allocation

$(4\alpha, 4(1-\alpha))$

$(4\beta, 4(1-\beta))$

lies in the core (this is the allocation that emerges from a perfect information equilibrium). Note that this allocation is compatible with information available within the economy. From example 2, we know that there is no REE for this economy. Further, for a choice of $\alpha \neq 1/2$, agent 1 gets a different allocation in the two states, and the above allocation cannot be supported as a non-revealing equilibrium (which would imply that agent 1 consumes the same amount in each state).

Suppose, therefore, that there exists a set of lump sum transfers, L_1 and L_2, with $L_1+L_2=0$, and an $\varepsilon\neq 0$ such that $p_1=1+\varepsilon$ is an equilibrium which supports the above allocation. From the market clearing conditions, one can easily check that the only solution, given $\alpha+\beta=3/4$ is $\varepsilon=0$, $L_1=0$, $L_2=0$.

Hsieh and Srivastava [1981] provide an example of an optimal allocation which cannot be decentralised even when there exist REE's. Figure 7 contains an example with non strictly-convex preferences.

However, core allocations can almost always be decentralized as REE's, in the sense of the the following Theorem.

Theorem 11 : Let $x \in C^D$, and suppose x cannot be decentralized as an REE. Then, there exists a sequence of economies, $((\geq)_n, k_n)$ such that $k_n \to 0$, $(\geq)_n \to (\geq)$ and such that x can be decentralized as an REE for all n.

Proof: Lemma 9 ensures that x is PO. A standard theorem then establishes that there exists a full information price vector, p, and a set of lump sum transfers, L^i, such that x is a full information competitive equilibrium. x cannot be decentralized, which implies that p must be non-revealing. Appealing to Theorem 3, we have that there exist $(\geq)_n \to (\geq)$, income perturbations $k_n \to 0$, and fully revealing prices $p_n \to p$ such that p_n is a full information equilibrium for the economy $((\geq)_n, k_n+L)$.

To conclude, we have seen that most econimies have an REE that lies in the core. If an economy has only an inefficient REE, then there is another economy, arbitrarily close by, which has an efficient REE. It is not clear whether most economies have only efficient REE's. We also know that if a core allocation cannot be decentralized as an REE, then there is an economy arbitrarily close by for which the same allocation lies in the core and can be decentralized.

This brings us to the question posed in the introduction. Is an REE a natural notion of equilibrium in the sense that it inherits the properties of a competitive equilibrium in the case of perfect information ? As far as existence and optimality properties are concerned, we believe the answer is yes, albeit in a generic sense.

VI. TRADING AFTER THE OCCURRENCE OF THE STATE

In this Chapter, we examine cooperative and non-cooperative outcomes when trading takes place after a state of nature occurs. As will be seen, some of the results of the previous sections change significantly. We shall discuss only the case of expected utility maximization.

First, we have to modify our notion of information. Previously, all that was meant by information was the ability to distinguish between states, while no mention was made of subjective probabilities. In fact, in our examples, we held such probabilities fixed. Once a state occurs, it is no longer possible to retain fixed subjective probabilities. Consider the representative agent, with initial information $I_o = \{E_1,\ldots,E_k\}$, the E_j being mutually exclusive events. Let $\pi_1,\ldots,\pi_k$ be the prior subjective probabilities of these events (i.e. $\pi_j = \Sigma\ \rho(s)$), and suppose, without loss of generality, that $s \in E_1$ occurs. The agent immediately knows that $E_2,\ldots,E_k$ have not occurred. His revised subjective probabilities, therefore, are $\pi'_1=1$, $\pi'_j=0$, $j=2,\ldots,k$. In the sequel, we shall assume that $s \in E_1^i$ occurs, $i=1,\ldots,m$. This entails no loss of generality. Further, we shall change terminology, and refer to the events E_1^i as 'information', without referring explicitly to the partition they belong to. We shall denote by $\rho^{i\prime}(s)$ the revised subjective probabilities of states in E_1^i, where

$$\rho^{i\prime}(s)=\rho^i(s)/\pi(\sim E_1^i) \text{ if } \pi(\sim E_1^i) \neq 0 \text{ and } s \in E_1^i$$

single relative price, p=1, and this is the only source of information to a2 besides his endowment.

Suppose next that S occurred. Once again, the same spot exchange of one umbrella for one bathing suit is possible, and a2 does not require any extra information to execute this trade. Again, there is only one relative price, p=1.

So far, we have seen that there need be only one market in this ex-post example, and the operative spot market, or 'market structure', is independent of which state of nature occurs[2]. This line of reasoning is very different from that of Radner [1979], who also studied the same problem. In his work, there are two relative prices to consider, p(R) and p(S). Then, $p(R) \neq p(S)$ reveals the state to agent a2, while p(R)=p(S) is non-revealing. If the analysis is truly ex-post, however, we have seen that there need be only one observable relative price, and the agent can therefore never compare p(R) and p(S). The only way in which p(R) and p(S) become relevant is if we have two markets, which puts us back in the contingent commodity setting, which is difficult to interpret in an ex-post model. The operation of only one spot market allows an optimum to be attained. This can be seen by supposing that both agents had perfect information to start with (by changing the endowment of a2 slightly, to say 2 U,3 U). Then, if R occurs, both

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[2] This type of argument is developed and expanded in Shefrin [1979]

$\rho^{i\prime}(s)=\rho^{i}(s)$ if $\pi(\sim E_1^i) = 0$ and $s \in E_1^i$,
$\rho^{i\prime}(s)=0$ otherwise.

Next, we discuss what is meant by market structure in this case, and examine the consumers problem. We shall argue that this ex-post case reduces essentially to that of a competitive equilibrium under uncertainty.

We start with an example. Suppose there are two agents, a1 and a2, and two possible states of nature, rain (R) and shine (S). Initial endowments are :

	state R	state S
agent a1	1 B	2 B
a2	2 U	2 U

where B = bathing suit, and U = umbrella

$I^{a1}=[\{R\},\{S\}]$

$I^{a2}=[\{R,S\}]$

Further, we suppose that R has occurred, that one umbrella provides adequate protection from the rain, and that one bathing suit is all that is required per person if sunny. On the basis of initial information, a1 knows that it is raining, while a2 does not.

Consider the spot exchange of one umbrella for one bathing suit. This trade is not contingent on any state of nature; a2 does not need to know which state has occurred in order to undertake this exchange, which we assume is desirable for both agents. There is a agents know this, and the perfect information equilibrium is the same as the spot market equilibrium, one bathing-suit being exchanged for one umbrella. The same holds if S occurs.

We turn next to the consumer's problem. Consider the representative agent with initial information E_1, and revised subjective probabilities $\rho'(s)$, where $\Sigma\rho'(s)=1$, with $\rho'(s)=0$ for $s \in \sim E_1$. The question at hand is the appropriate objective function of the agent. The agent knows that $s \in E_1$ has occurred, and is interested in current consumption. If we stick to our spot market model, then if his net trade on the spot market is $z \in R^L$, his realised utility is $U(e+z)$, where e is his endowment in the event E_1. The 'measurability' of e with respect to initial information implies that $e(s)=e$ for all s in E_1. As such, therefore, which state in E_1 actually occurs is irrelevant to him, and he can simply choose z to maximise $U(e+z)$, which is the utility he is going to get. This problem is exactly the same as the standard model of competitive equilibrium with certainty. Not surprisingly, equilibrium allocations allocations for this economy lie in the core.

So far, we have adopted Shefrin's way of modelling market structure in the ex-post case[3]. The above interpretation of consumer behavior in this case, however, is different from his. Even after the occurrence of the state, Shefrin requires the agent to maximise

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[3] See Shefrin [1979]

expected utility, not realised or actual utility. The agent still maximises $\Sigma\rho(s)U(x(s))$. As there is only one spot market, the competitive allocation turns out to be inefficient, since there is no possibility of inter-state trading. He concludes that spot trading eliminates the inefficiency arising from the informational constraint in the ex-ante case, but that the lack of trading across states still leads to inefficiency. Strictly speaking, our model is not directly comparable to that of Shefrin, since it is not clear whether he compares equilibria to others attainable with the information held by agents within the economy.

We have argued that ex-post, only the realised or actual utility of the agent matters. In this case, there is no need to trade across states ex-post, since the only concern of the agent is the utility he receives in the event E_1. The conclusion, then, is that if we accept both spot trading and the maximisation of realised utility, then the ex-post case reduces to that of a competitive equilibrium under certainty, and there is no inefficiency associated with equilibrium. Below, we shall show that if we let agents maximise expected utility, but impose the constraint that final allocations be measurable with respect to final information, then the resulting equilibrium is the same as the spot market equilibrium with agents maximising realised, or actual, utility. The model of the previous sections can be easily modified to formalise these concepts. In fact, one can allow for very general forms of communication even in the non-cooperative case. We shall do this in a while, but first, we examine similar issues in relation to the core.

Definition 15 : A viable coalition consists of a set of agents $C\subset\{1,\ldots,m\}$, a set of allocations, $(\sigma^i(s))$, and a set of final informations, (A^i), such that :

(i) $A^i \subset E_1^i$, all $i \in C$

(ii) $\sigma^i(s_1)=\sigma^i(s_2)$ for s_1, $s_2 \in A^i$

(iii) $\Sigma\sigma^i(s) \leq \Sigma\, e^i(s)$

(iv) $\Sigma\rho^{i}{}'(s)U^i(x^i(s)) \geq \Sigma\rho^i(s)U^i(e^i(s))$

Condition (i) requires that final information be no worse than initial information, while (ii) is the measurability constraint, as in Wilson [1976]. We remark that this definition is somewhat contorted, since the measurability constraint implies that the allocation across states is the same, and we therefore do not need to consider allocations in different states. It is retained in the above form for ease of comparison later on.

Definition 16 : An allocation-information list (y^i,B^i) is said to lie in the core if :

(i) $B^i \supset \cap E_1^i$, $B^i \subset E_1^i$, all i

(ii) $y^i(s_1)=y^i(s_2)$ for s_1, $s_2 \in B^i$

(iii) (y^i,B^i) is not blocked by a viable coalition, with the obvious definition of blocking.

Definition 17 :

(a) The coarse core consists of allocations not blocked by a viable coalition with $B^i = E^i_1$.

(b) The fine core consists of allocations in the coarse core not blocked by a viable coalition with information $B^i = \cap E^i_1$, all $i \in C$.

Theorem 12: The fine core is non-empty.

Proof : Without loss of generality, suppose $s \in E^i_1$ occurs, all i. Then, $e^i(s_1)=e^i(s_2)$ for $s_1, s_2 \in E^i_1$, all i. Then, consider the economy with no uncertainty, in which the initial endowment of agent i is $e^i=e^i(s)$, $s \in E^i_1$, and the utility function of the agent is $U^i(x)$. The core of this constructed economy is non-empty by standard theorems, so let (y^i) be such a core allocation. Define $\sigma^i(s)=y^i$ for $s \in E^i_1$, and let $E = \cap E^i_1$. Then, $\sigma^i(s)$, $s \in E$, is unblocked for the economy with full information, as :

Suppose not. Then, there exists a coalition C and an alternative allocation-information set, say (x^i, A^i) such that

$$V^i(x^i) \geq V^i(\sigma^i),\ i \in C, \text{ with}$$
$$V^i(x^i) > V^i(\sigma^i) \text{ for some } i \in C.$$

Note that for $s_1, s_2 \in A^i$, $x^i(s_1)=x^i(s_2)$ by the measurability of the final allocation. Further, since $\Sigma_{s \in A^i} \rho^{i,}(s)=1$, $V^i(x^i) = U^i(x^i)$, with the obvious notation. But this implies that

$$U^i(x^i) > U^i(y^i)$$

which means that (x^i) dominates (y^i) for the constructed economy, a contradiction.

Thus, retaining the measurability assumption of Wilson, and imposing our own requirement that agents be smart enough to distinguish between states on the basis of endowments, we find that there is no existence problem. Next, we show that if we extend the spot market interpretation to the core, and let agents worry about realised utility, then the resulting allocations are the same as if we retain the measurability assumption. In this sense, this assumption is non-binding.

Definition 18 :

(a) A spot coalition consists of a set of agents, $C \subset \{1, \ldots, m\}$, and a set of allocations (σ^i), such that

(i) $\Sigma \sigma^i \leq \Sigma\, e^i$, where $e^i = e^i(s)$, $s \in E_1^i$

(ii) $U^i(\sigma^i) \geq U^i(e^i)$, all $i \in C$.

(b) The spot-core (S-core) consists of allocations not blocked by any spot coalition.

The following is well known :

Theorem 13: The S-core is non-empty.

Theorem 14: The viable core and the S-core are equivalent.

Proof : This is seen by inspection. The measurability constraint immediately implies that any allocation attainable by a spot coalition is also attainable by a viable coalition, and conversely. The conclusion follows directly.

Corollary : The viable core is non-empty.

The same equivalence extends to an REE defined with measurability and a spot market equilibrium.

Definition 19 : A Rational Expectations Equilibrium (REE) consists of a set $A \subset S$, $p(s) \varepsilon R^L_+$, $s \varepsilon A$, and a set of allocations $\{x^i(s)\}$, $i=1,\dots,m$, $s \varepsilon A$, and a set of events $A^1,\dots,A^m$ such that :
(i) $A = \cap A^i$, $A^i \subset E^i_1$, all i.

(ii) For each i, prob$(s \varepsilon A^i)=1$.

(iii) For s_1, s_2 εA, $p(s_1)=p(s_2)$.

(iv) $p(s) = 0$ for all $s \varepsilon \sim A$

(v) $x^i(s)$ maximises $\Sigma \rho^{i'}(s) U^i(x^i(s))$ subject to the budget constraint and the constraint that s_1, s_2 $\varepsilon A^i \Rightarrow x^i(s_1)=x^i(s_2)$.
(vi) $\Sigma_i x^i(s) \le \Sigma_i e^i(s)$, $s \varepsilon A$.

A is the maximal event in which trading takes place, while A^i is the minimal event that agent i knows has occurred. (i) requires that final information, as represented by A^i be at least as good as initial information, (ii) is a consistency requirement on subjective probabilities, while (iii) is the 'measurability' constraint on prices, requiring that expectations be fulfilled. For convenience, we set prices to zero in all states in which no trading occurs. (v) requires each agent to behave optimally, while (vi) is market clearing. There is no restriction on the A^i besides (i). The definition of an REE thus allows for very general forms of communication between agents.

Definition 20 : A spot market equilibrium is a price vector, $p \in R^L_+$, and a set of allocations, (x^i), $i=1,\ldots,m$ such that :
(i) $\Sigma\, x^i \leq \Sigma\, e^i$, $e^i = e^i(s)$ for $s \in E^i_1$
(ii) x^i maximises $U^i(x^i)$ subject to $p.x^i \leq p.e^i$.

We state :

Theorem 15: There exists an S-equilibrium.

Theorem 16: An S-equilibrium is an REE, and conversely.

Corollary : The exists an REE.

In the ex-post case, therefore, an REE lies in the core, which follows from the spot-market equilibrium lying in the S-core. It is easy to apply standard theorems to show that any core allocation can also be decentralised as an REE for a given set of initial informations.

Finally, we discuss how our analysis is related to the efficient markets hypothesis. This hypothesis is generally described as maintaining that the market price embodies all the information held by agents within the economy. Our ex-post model is certainly compatible with this view, and goes a little further. Information in the model consists of being able to tell which state of nature has occurred. The market price does depend on this information. The resulting allocation is efficient in the sense that it lies in the core. The market price does not reveal the state of nature in general, but this is seen to be irrelevant, since the spot market allocation coincides with the perfect information equilibrium.

VII. LARGE ECONOMIES

In this Chapter, we demonstrate that our results are generally invariant to the size of the economy. To achieve a contrast, we shall examine the opposite extreme of the model examined so far, namely one with a non-atomic measure space of consumers. The model is adapted from Hildenbrand [1974].

Let $A=[0,1]$, and let ν be a non-atomic probability measure on $(A,B(A))$, where $B(A)$ is the Borel σ-algebra of A. An element $C \in B(A)$ is a coalition, while A is the set of consumers. The initial information of $a \in A$ is given by I^a_o, and his final information is denoted by I^a. The meet of I^a, $a \in C \in B(A)$ is written as VI^a, $a \in C$. The definitions of final information, REE, a coalition, the core, etc. carry over from the previous chapters. The only difference is that a coalition can only consist of sets in $B(A)$, and that statements that previously held for $i=1,\dots,m$ must now hold for all $a \in A$. The translation of definitions is direct, and so we do not state them explicitly.

The assumption that preferences are convex can be dropped. Preferences, $(\geq)_a$, have to satisfy the technical requirement that for $x,y \in X$, $\{ a : x(\geq)_a y\}$ is measurable.

Let $\phi((\geq),k)$ denote the Walrasian, or full information, equilibrium correspondence. The following is well known :

<u>Theorem 17 :</u> $\phi((\geq),k) \neq \emptyset$ $C^P \neq \emptyset$

Again, an REE need not exist. Example 2 of Chapter III can be modified to demonstrate this, as follows:

Example 11 :

Let ν be the Lebesgue measure on A. Initial data is:

	state		
	s_1	s_2	
agent a	2	2	$a \in [0,1/2)$
b	1	3	$b \in [1/2,1]$

$I^1 = [\{a,b\}]$

$I^2 = [\{a\},\{b\}]$

$\rho^a = (\alpha, 1-\alpha)$ if $a \in [0,1/2)$,

$\rho^a = (\beta, 1-\beta)$ if $a \in [1/2,1]$

$u^a(x) = \log(x)$, all a.

Agents are thus partitioned into two groups with equal weight. The first group is identical to consumer 1 in Example 2, the second to consumer 2. It is straightforward to verify that exactly the same argument holds here as in Example 2, i.e. there does not exist an REE for this economy.

REE's do exist generically, and to adapt the proof of Theorem 3, we need to specify the economy somewhat more precisely. Formally, an economy is a map $\Xi:(A,B(A),\nu) \to P \times R$, where P is the space of preferences and R is the space of income perturbations. $k(a)$ denotes the income perturbation of agent a.

Theorem 18 : Suppose Ξ does not have an REE. Then, there exist $(\geq)^n_n, k_n(a)$ such that Ξ_n has an REE for each n, and such that $(\geq)^n_a \to (\geq)_a$, all a, and $k_n(a) \to 0$, all a.

Proof: The construction in the proof of Theorem 3 is done pointwise, and the proof therefore follows directly from Theorem 3.

Note that we have implicitly required pointwise convergence in the above theorem. This can be made more appealing by examining economies as distributions, rather than via the map Ξ. A formal presentation of this equivalent representation is contained in Hildenbrand [1974], and we do not repeat it here.

Example 8 can be extended in the same way as above to show that an REE need not lie in the core. The existence of core allocations is also immediate. As in Theorem 5, it is easy to see that the constructed "perfect information" economy has a non-empty core, and that allocations that lie in the core of the constructed economy also lie in the core of the original economy. We state:

Theorem 19: The core is non-empty.

Corollary: The coarse core is non-empty, as is the fine core.

The relationship between C^D and C^P is exactly the same in the continuum economy as it is in the finite economy. This may seem surprising in the light of Example 6, since we would expect the monopoly power of an agent with better information to lessen with an increase in the number of agents, at least if aggregate initial information is perfect. This is not true, the reason being that even with a continuum of agents, the core need not coincide with the set of all PO allocations. Thus, it is possible to construct an allocation in C^D which is not in C^A. In fact, this result requires only one agent with superior information, even though the measure ν is non-atomic. This can be seen by partitioning A into three subsets, $A_1=[0,1/2)$, $A_2=[1/2,1)$, $A_3=\{1\}$, with each A_i corresponding to agent i in example 6. Then, in the perfect information economy, it is possible for coalitions A_1 and A_2 to collude against A_3, but not in the economy with differential information.

It is easy to see that Example 9 can also be extended to hold in the large economy, i.e. that there exist Pareto Optimal REE's which do not lie in the core.

Finally, it is true that "most" core allocations can be decentralized in the sense of Theorem 11. This follows from Theorem 18 combined with Theorem 3 of Hildenbrand [1968]. The reasoning is the same as that in the economy with a finite number of consumers.

VIII. ENFORCEABILITY OF CONTRACTS

In this Chapter, we examine the important issue of the enforceability of contracts. In the ex-ante model of Chapters II-V, all contracts were made prior to the occurrence of the state. It was assumed that such contracts would be binding, i.e. that when a state of nature actually occurred, agents would not attempt to renege on their commitments.

In general, it is usually in the interest of some agents to recontract after a state of nature occurs. The following example illustrates.

Example 12 :

Consider an economy with two agents, three states and one commodity. Initial information and endowments are given below.

	state a	state b
agent 1	2	2
2	1	3

$I^1=[\{a,b\}]$

$I^2=[\{a\},\{b\}]$

$U^1(x) = U^2(x) = \log(x)$; the subjective probabilities of the agents are $(\alpha,1-\alpha)$, $(\beta,1-\beta)$.

Then, σ with

$\sigma^1=(4\alpha,4(1-\alpha))$

$\sigma^2=(4\beta,4(1-\beta))$

lies in the core.

Suppose that this contract is agreed upon, and now state b occurs. If α is such that $4(1-\alpha)<2$, i.e. $\alpha>1/2$, then adhering to the contract gives agent 1 lower utility than if what he can get by refusing to trade, i.e. by breaking the contract. Note that if agent 1 refuses to trade, he does so at the expense of agent 2. Thus, agent 2 has an incentive to enforce the contract.

The illustration provided by this example is not specific to an economy with differential information. It applies equally well to an economy in which all agents have perfect information. In what follows, we shall examine which of the contracts that arise in our model are enforceable. We shall only consider the case of expected utility maximization.

The example motivates a definition of enforceability. We say that a contract is enforceable if at least one of the parties has an incentive to sue for breach of contract. One can imagine a legal system which would arise for the purpose of implementing such litigation. This definition of enforceability can be rephrased as follows. We can say that a contract is enforceable if for any re-allocation of resources ex-post, some agent is made worse off. This says that ex-post, the contract must be Pareto optimal:

Definition 21 : A contract $x=(x^i(s))$, $i=1,\ldots,m$, $s=s_1,\ldots,s_k$ is enforceable in s if there is no allocation $(y^i(s))$, $i=1,\ldots,m$ such that

(i) $\Sigma y^i(s)\leq\Sigma e^i(s)$

(ii) $U^i(y^i(s))\geq U^i(x^i(s))$, all i, and

$U^i(y^i(s))>U^i(x^i(s))$, some i.

If x is enforceable in all $s\epsilon S$, then x is called enforceable.

The definition states that once the state has occurred, then the agreements in x which are contingent on s be ex-post Pareto optimal. If they are not, then there is no incentive for any agent to stick to the terms of the contract, since renegotiation can make everyone better off. Thus, any attempt to renege on an enforceable contract will be opposed by at least one agent, and for this reason, we call such contracts enforceable.

Theorem 20 : Core allocations are enforceable.

Proof: Let (x,I) be a core allocation, and say $s_1\epsilon S$ occurs. Without loss of generality, suppose x is not enforceable in s_1. Then, there exists an allocation y, with

$U^i(y^i(s_1))\geq U^i(x^i(s_1))$, all i, and with strict inequality holding for some i.

Consider first the case in which aggregate initial information is perfect. In this case, consider the allocation

$$\sigma^i(s_1)=y^i(s_1)$$

$\sigma^i(s)=x^i(s)$ for all $s\neq s_1$.

Since aggregate initial information is perfect, (σ, I_o^i) is physically and informationally feasible for the grand coalition. Also,

$$V^i(\sigma^i) = \Sigma p^i(s)U^i(\sigma^i(s))$$

$$=p^i(s_1)U^i(y^i(s_1))+\Sigma p^i(s)U^i(x^i(s)),$$

(the summation being over $s\neq s_1$)

$$\geq V^i(x^i), \text{ all } i, \text{ and}$$

$$>V^i(x^i), \text{ some } i.$$

This implies that (x,I) is blocked by the grand coalition with the allocation information pair (σ, VI_o^i).

If aggregate initial information is not perfect, then let A be the event in VI_o^i such that $s_1\epsilon A$. Then, let $\sigma^i(s)=y^i(s_1)$ if $s\epsilon A$, and $\sigma^i(s)=x^i(s)$ otherwise. The same proof follows.

<u>Corollary 1:</u> Any Pareto Optimal allocation is enforceable.

This implies that there are enforceable contracts that do not lie in the core. Example 6 provides one such example.

<u>Corollary 2:</u> Fully revealing REE's are enforceable.

Together with Example 10, we get the following somewhat surprising result.

Theorem 21 : There exist enforceable, non-revealing REE's which do not lie in the core.

We conclude that enforceability is straightforward as far as core allocations are concerned. This is not true with REE's. Example 8 contains an example of an REE that does not lie in the core. It is clear that this REE is not enforceable, since there exists an ex-post allocation which makes both agents better off. In fact, the allocation in Example 8 is not even Pareto Optimal. On the other hand, fully revealing REE's are enforceable. So are Pareto Optimal REE's, even though they may be non-revealing and not lie in the core.

The main conclusion of this section is that REE's as in Example 8 will not be enforceable. All parties to the contract may be better off from renegotiation of the contract. This provides some motivation for the existence of both spot and futures markets. Spot trading becomes useful, unlike the Arrow-Debreu paradigm, when, due to the presence of differential information, there are gains to be had from ex-post trading which simply do not exist in the futures market. In particular, no non-Pareto optimal contract will be enforceable, and in such cases, one would almost certainly observe spot trades taking place after the state of nature gets revealed. It is often argued that the Arrow-Debreu representation of an economy is misleading in the sense that it cannot explain the simultaneous existence of both spot and futures markets. We have seen that even in the absence of an explicitly dynamic model, differential information can provide a role for such a dual existence.

FIGURES and REFERENCES

Figure 1

The topology of closed convergence

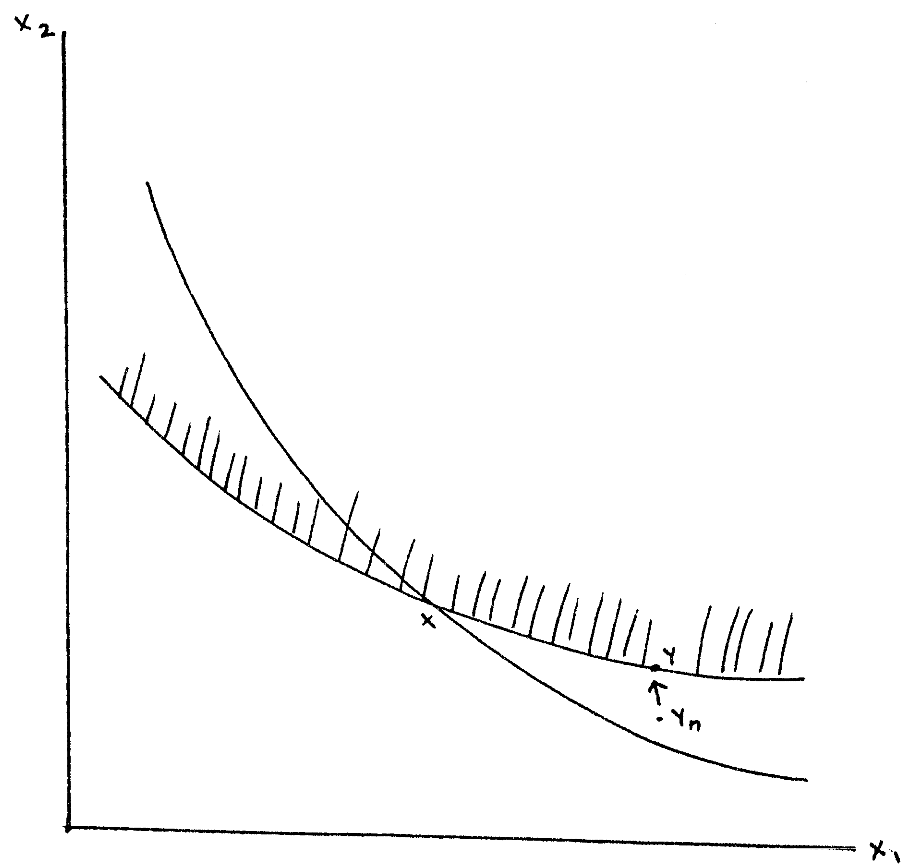


Figure 2

The budget correspondence

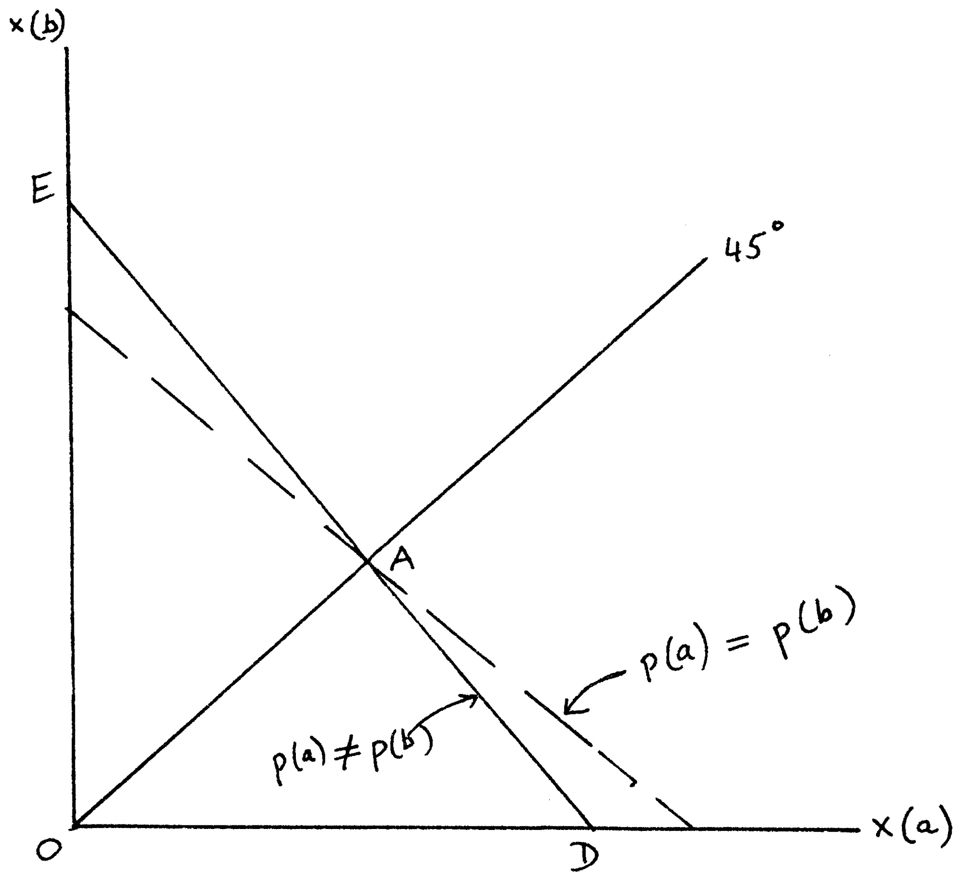


Figure 3

The non-existence of an REE

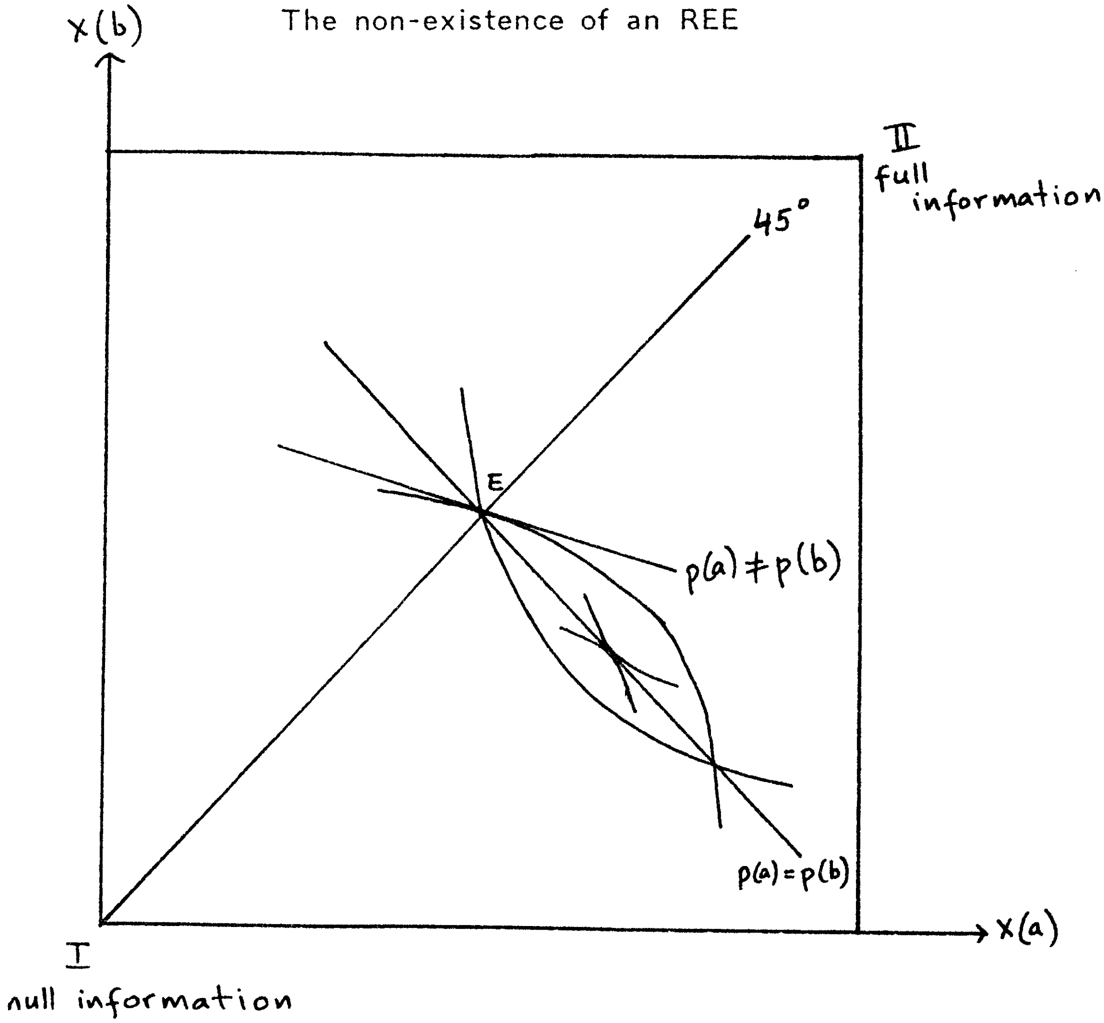


Figure 4

The price space

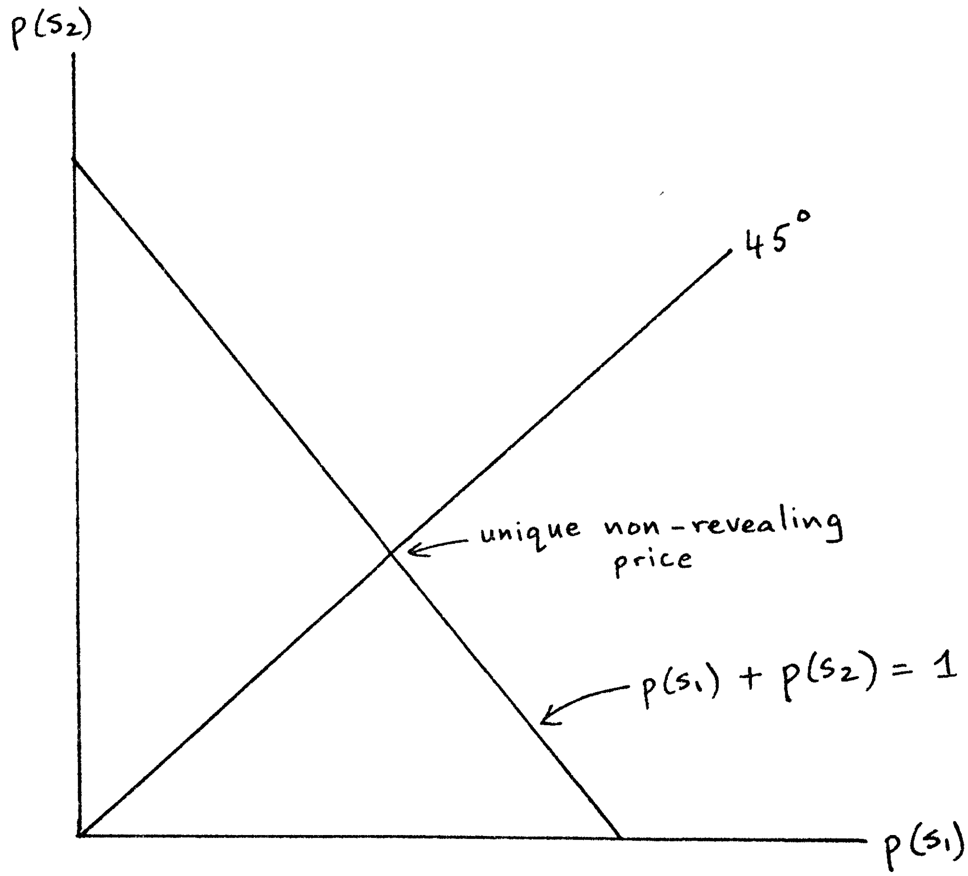


Figure 5

Generic Existence

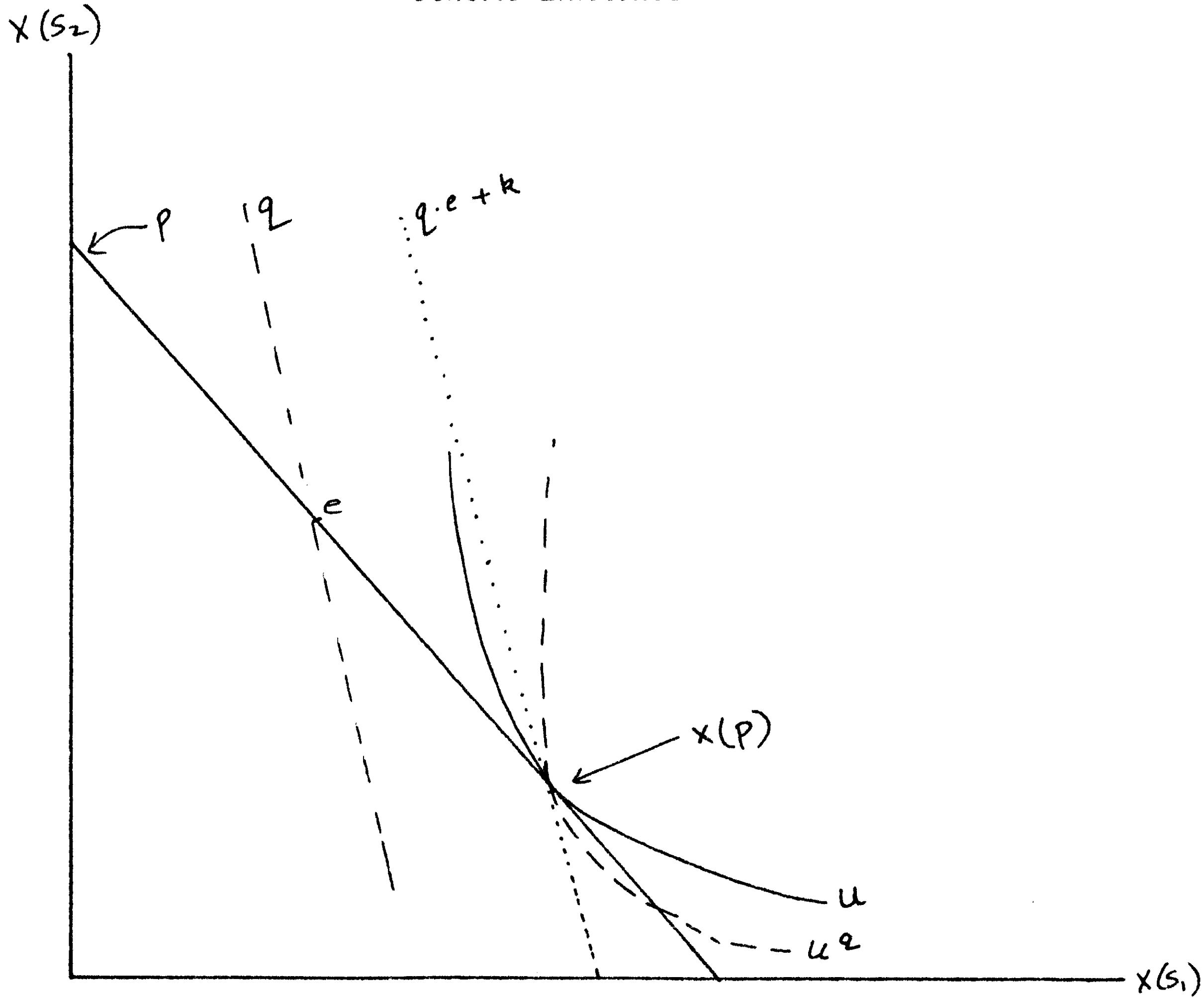


Figure 6

Inefficient Rational Expectations Equilibria

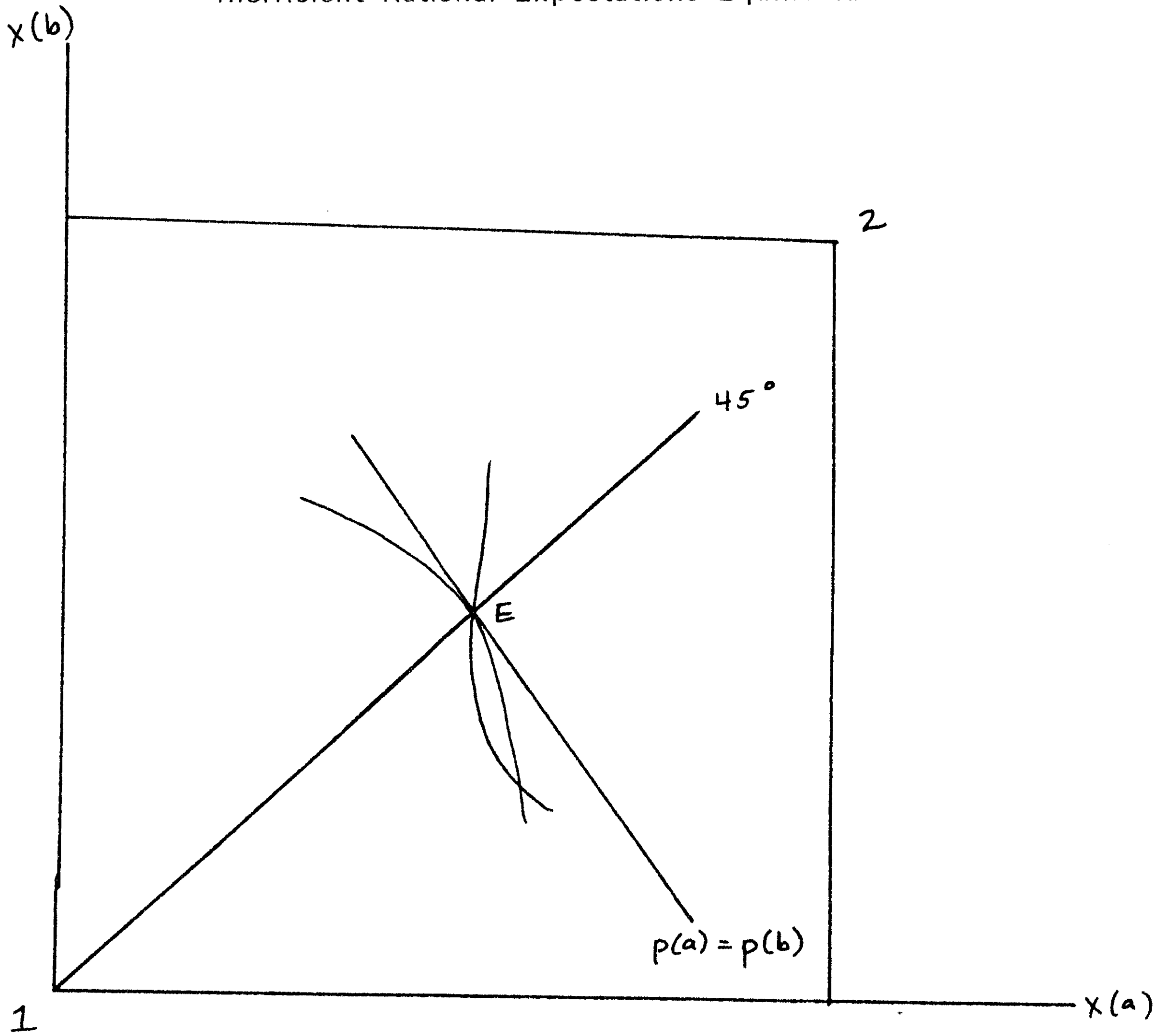
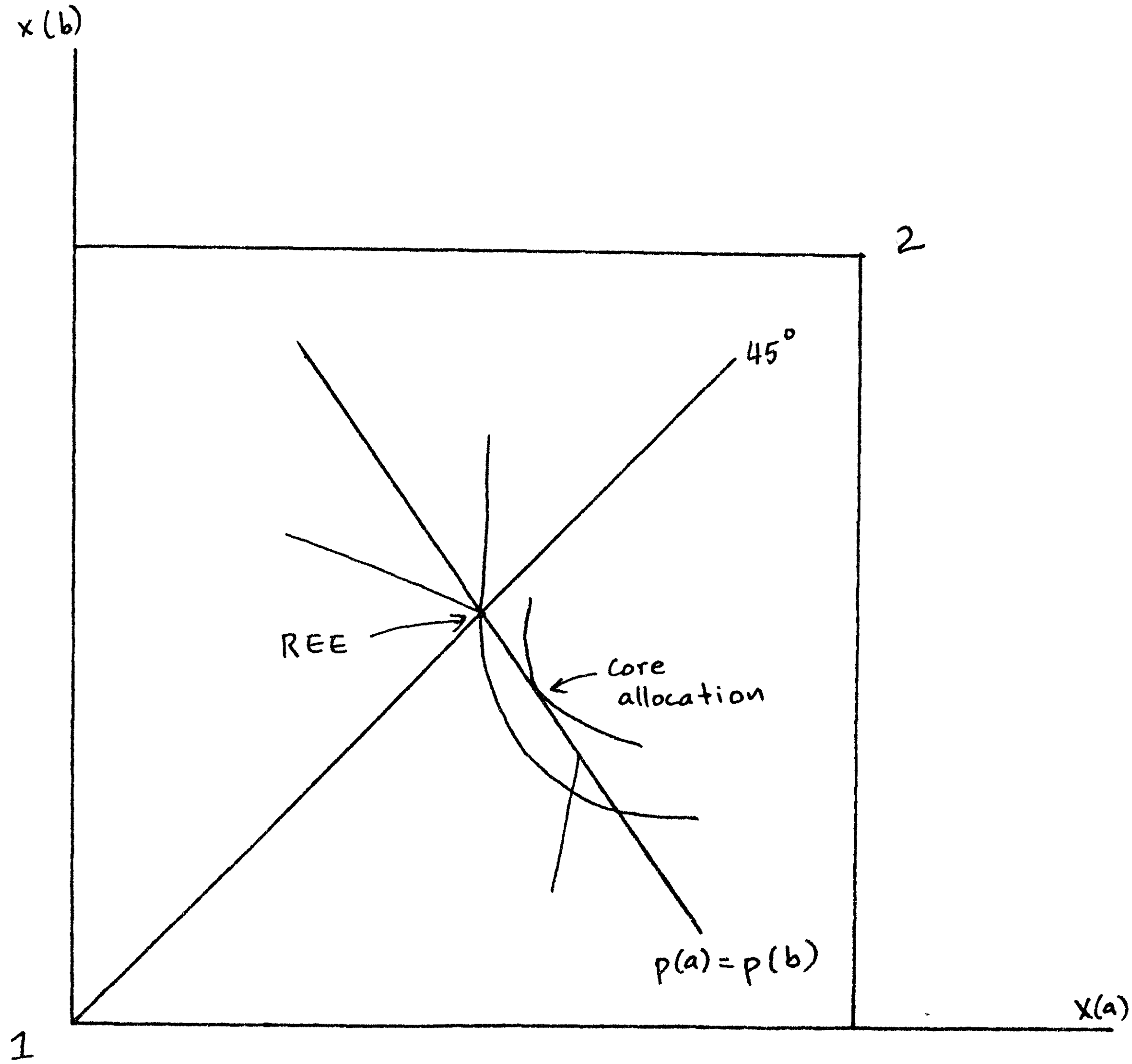


Figure 7

Inefficient Rational Expectations Equilibria



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